Hierarchical Economic Dispatch for Piecewise Quadratic Cost Functions

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ABSTRACT-- This paper presents a method to solve the economic power dispatch problem with piecewise quadratic cost functions. The solution approach is hierarchical, which allows for decentralized computations. An advantage of this approach is the capability to optimize over a greater variety of operating conditions. Traditionally, one cost function for each generator is assumed. In this formulation multiple intersecting cost functions are assumed. This method has application to fossil generation units capable of burning gas and oil, as well as other problems which result in multiple intersecting cost curves for a particular unit. The results show that the solution method is practical and valid for real-time application. The motivation for this research stems from the actual operational and planning problems of a large Southwestern Utility.

INTRODUCTION

Present operating conditions of many generation units within utilities require that the generation cost functions for fossil fired generation be segmented as piecewise quadratic functions. The reasons for this partitioning of the cost curves are varied. Often this is done to increase the accuracy of the functional relationship. More recently, a reason for segmenting cost functions results from multiple fuel sources for each generation unit. Some generation units, especially those units which are supplied with numerous sources (gas and oil) of fuel, are faced with the dilemma of determining which fuel is most economical to burn. In general, the input-output relationship for output megawatts (MW) versus the number of BTU's input will be an approximate quadratic function. Conventionally, a single input-output relationship was multiplied by the cost per BTU of fuel to determine a functional relationship for operating cost.

Presently, the capability of burning gas or oil at a single unit poses the problem of at least two cost curves for a single unit. In general, these curves are not parallel. Intersecting curves implies that it may be more efficient to burn oil for some MW outputs and natural gas for others. Additionally, varying heat contents of natural gas from multiple suppliers could result in cost curves which are not parallel when compared to each other. Presently, a major Southwestern Utility must address the problem of up to three different sources of natural gas and one source of oil feeding each generation unit simultaneously. This problem requires a more general approach to economic dispatch.

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The notion of multiple cost curves is not limited to applications with multiple fuels. It is possible to generalize this approach to consider multiple cost curves for units to represent units operating under less than fully operational condition. Mechanical failures will give rise to variations in the cost functions which, in general, will yield intersecting families of cost curves. As the cost of fossil fuels increases, better modeling of the actual input-output relationship for each generation unit is essential.

The approach pursued is hierarchical. This has several advantages [5]. First, the decentralized configuration of the generation facilities of most power systems lends itself to a decentralized approach. Each plant in the system is most likely to be aware of the various availabilities of fuel, and or operational condition of the units. Hence, the plant is best able to determine which cost curves associated with which unit are currently applicable. Further, since in this more generalized approach, each cost curve carries the further significance that it may represent different operational characteristics such as switching fuels or completing repairs, it becomes more convenient to decentralize determination of cost curves to the plant level.

For any given unit with multiple cost curves, the curves can be superimposed as shown in Figure 1. By inspection, to minimize cost, it is necessary to operate each unit on the lower contour of the intersecting curves. The resulting cost function is termed a "hybrid cost" function. This hybrid cost function is piecewise quadratic. Unlike conventional piecewise quadratic cost functions, each segment of the hybrid cost function implies some information about the type of fuel being burned or the operational characteristics of the unit.

The second reason which motivates a hierarchical approach is computational efficiency. With each unit having multiple quadratic segments to represent cost, keeping track of which segment each unit is operating on would computationally overburden conventional centralized approaches to economic dispatch. The efficiency of the hierarchical approach is realized for both hybrid cost curves and conventional piecewise quadratic cost curves.

PROBLEM FORMULATION

In this problem, the piecewise quadratic function is used to represent multiple fuels which are available to each generation unit. The hybrid cost function and hybrid incremental cost function of unit j in subsystem i is shown in Figure 1. These functions are defined as

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$$\text{COST}(P_{ij}) = \begin{cases} A_{ij1}+B_{ij1}*P_{ij}+C_{ij1}*P_{ij}^2, \text{ fuel 1}, \\ \qquad P_{ij(min)} \le P_{ij} \le P_1, \\ A_{ij2}+B_{ij2}*P_{ij}+C_{ij2}*P_{ij}^2, \text{ fuel 2}, \\ \qquad P_1 \le P_{ij} \le P_2, \\ \vdots \\ A_{ijk}+B_{ijk}*P_{ij}+C_{ijk}*P_{ij}^2, \text{ fuel k}, \\ \qquad P_{k-1} \le P_{ij} \le P_{ij(max)}. \end{cases} \quad (1)$$

$$\text{INCS}(P_{ij}) = \begin{cases} B_{ij1}+2*C_{ij1}*P_{ij}, \text{ fuel 1}, \\ B_{ij2}+2*C_{ij2}*P_{ij}, \text{ fuel 2}, \\ \vdots \\ B_{ijk}+2*C_{ijk}*P_{ij}, \text{ fuel k}. \end{cases} \quad (2)$$

where A_{ijk}, B_{ijk}, C_{ijk} are cost coefficients of fuel k. Subscript i indicates subsystems, subscript j indicates units, and subscript k indicates fuel type. The hybrid cost functions give rise to an additional variable, f, which describes the available fuels. The Lagrangian with the transmission loss term neglected is written as,

$$L(f,p,\lambda) = F(f,p) + \lambda' G(p) \quad (3)$$

where

f = discrete variable which indicates the fuel associated with each segment of the hybrid cost function as shown in Figure 1, changes in f indicate switching between fuels,
λ = Lagrangian multiplier, or incremental cost,
p = control vector of power,
F(f,p) = total cost,
G(p) = power balance constraint, (demand - generation),
' = transpose.

The hierarchical structure of the power system is composed of several subsystems (power plants). Each subsystem includes several generation units as shown in Figure 2. The power outflow from each subsystem is referred to as the subsystem demand. Generally, the power balance constraints of the system and the subsystems are written as:

$$G(p) = 0 \qquad \text{for the system,}$$

$$G_i(p_i) = 0 \qquad \text{for subsystem i.} \quad (4)$$

In Equation 3, the additional control variable f complicates conventional approaches to determine the optimal solution. This approach considers the fuel variable, f, separately from other control parameters. The solution strategy must select an operating fuel for initialization in the subsystem dispatch. An equivalent cost function [4] can be calculated from the cost coefficients of the chosen fuels of all units in a subsystem, to represent this subsystem with current operating conditions. The fuel variable is eliminated from Equation 3 by utilizing the equivalent cost functions with the limitation that these equivalent cost functions are only valid for a certain range of power generation around the present operating point. These ranges of operation change when the operating fuel is changed.

The Lagrangian functions of all subsystems and the system can now be written as

$$L_i(f,p_i,\lambda_i) = F_i(f,p_i) + \lambda_i' G_i(p_i) \quad (5)$$

and

$$L(p,\lambda) = F(p) + \lambda' G(p). \quad (6)$$

The structure of this Lagrangian function is similar to other power system optimization problems [1,4]. Hence, the necessary conditions for an optimum (system) are

$$\frac{\partial L}{\partial p_i} = 0 = \frac{\partial F}{\partial p_i} + \left(\frac{\partial G}{\partial p_i}\right)' \quad (7)$$

$$\frac{\partial L}{\partial \lambda} = 0 = G(p) \quad (8)$$

Equation 7 can be written as,

$$\lambda = \frac{\partial F_i}{\partial p_i} \quad (9)$$

For each subsystem, the necessary conditions for an optimum are,

$$\frac{\partial L_i}{\partial f} = 0 \quad (10)$$

$$\frac{\partial L_i}{\partial p_{ij}} = 0 = \frac{\partial F_i}{\partial p_{ij}} + \left(\frac{\partial G_i}{\partial p_{ij}}\right)' \lambda_i \quad (11)$$

$$\frac{\partial L_i}{\partial \lambda_i} = 0 = G_i(p_i) \quad (12)$$

Equation 11 can be written as,

$$\lambda_i = \frac{\partial F_{ij}}{\partial p_{ij}} \quad (13)$$

Solution of Equation 10 requires more information than in the conventional approach to economic dispatch. It is necessary to know which segment of the cost function, or fuel type, results in minimum cost. Initially, a particular segment (fuel type) is chosen as a starting point. The choice of which segment to start on, will have an effect on the rate of convergence. A computational procedure for determining a "good" initial value is described.

INITIALIZATION

The "mean distribution" method (Appendix A) is introduced to determine the initial power by equally dividing the subsystem demand to all generation units according to their capacities.

$$P_{ij} = p(\text{demand}) \times \frac{P_{cij}}{P_{cs}} \quad (14)$$

where P_{ij} is the initial generation power of unit j, P_{Cij} is the capacity of unit j, P_{CS} is the capacity of this subsystem, p(demand) is the initial subsystem demand. An equation similar to Equation 14 is used to calculate initial values of subsystem demand from system demand. From Figure 1, the incremental cost of each generation unit can be calculated to burn a certain fuel k for a specified MW output power. In a subsystem, there may be several generation units in operation. The range of feasible solutions to the optimal incremental cost is bounded between the greatest value and the smallest value of the unit incremental costs determined by the mean distribution method (Appendix B).

SOLUTION ALGORITHM

A binary search [3,6] is applicable in searching for the optimal subsystem incremental cost. The upper bound and the lower bound of incremental cost of each subsystem are those greatest and smallest values of initial values determined from the mean distribution method. Generating power and operational fuel type, for each unit, may be changed during optimal dispatch. Dispatch techniques in subsystems associated with Equations 10, 12, 13 are shown more clearly in the flow chart of Figure 3. Convergence of each subsystem iteration is determined when generation and demand have an error within a tolerance of .01 percent. The power balance constraint must have a mismatch such that

$$G_i(p_i) \leq (\text{tolerance}) \times P(\text{demand}). \qquad (15)$$

The subsystem optimization will determine an optimal operating condition of each subsystem. The fuel variable f and the subsystem incremental cost are fixed after this evaluation. The equivalent cost function [4, page 126] of this subsystem can be calculated from the coefficients of the piecewise sections of generation units for a specific demand power. The equivalent coefficients A_e, B_e, C_e, are

$$C_e = \frac{1}{\sum_j \left(\frac{1}{C_{ijk}}\right)}$$

$$B_e = C_e \times \sum_j \left(\frac{B_{ijk}}{C_{ijk}}\right)$$

$$A_e = \sum_j \left(A_{ijk} - \frac{B_{ijk}^2}{(4 \times C_{ijk})}\right) + \frac{B_e^2}{(4 \times C_e)} \qquad (16)$$

where the fuel variable is included. The system dispatch is determined from these equivalent costs by assuming they are continuous from minimum generation to maximum generation of all subsystems. Actually, the equivalent cost is only accurate within a certain range near the present subsystem demand from which performance of this subsystem is optimized. If a further determination of subsystem demand goes beyond this range, this equivalent cost function is used to find a quasi-optimal condition. Differences between actual values and temporary values are accepted as iteration errors. A new equivalent cost function is calculated resulting from the generating power adjustments from the previous system dispatch or an operating section change (fuel change) resulting from a recent subsystem dispatch. Before the algorithm converges to a system incremental cost, and after each iteration of system dispatch, a target of system incremental costs and expected subsystem demands is calculated for the next iteration.

An algorithm for solving the optimization of the subsystems and the system described above is given as follows:

Step 1. Read in system information.

Step 2. Determine the hybrid cost functions and hybrid incremental cost functions and required data of all generation units in the system.

Step 3. Initialize generating power of all units in each subsystem from present system demand by the mean distribution method. Determine the operating fuel of each unit from known data of step 2.

Step 4. Run subsystem dispatch with known demands utilizing a binary search; check operating fuel of each unit, calculate subsystem incremental cost and generating power of each unit.

Step 5. Calculate equivalent cost of each subsystem after optimization. Go to step 4 unless all subsystems are completed, otherwise go to step 6.

Step 6. Run system dispatch utilizing all subsystem optimality conditions. A binary search algorithm is utilized to calculate the target of system incremental cost λ and expected subsystem demands. If the differences between the resultant subsystem and system demand requirements are all less than

$$((\text{tolerance}) \times p(\text{demand})),$$

then the system incremental cost represents the solution; if not go to step 4.

A detailed description of the dispatch procedure is shown in the flow chart of Figure 3.

EXAMPLE

A system with three subsystems and ten generation units is studied with several system demands. System characteristics are shown in Table 1 in which, generation (MIN) and (MAX) are the upper limit and the lower limit of each generation unit. There are three different kinds of fuels 1,2,3. Coefficients A,B,C are formulated following the example of Equation 1 and Figure 1. Unit 9 in this example is a special case where fuel 2 is not always economical to burn, but since fuel 2 is still available to this generation unit, if fuel 1 or 3 is exhausted or not available, fuel 2 may be substituted in the solution algorithm immediately. The cost function coefficients are, A:(MBTU/Hr), B:((MBTU/Hr)/MW), C:((MBTU/Hr)/MW^2), plus a constant term (MBTU = MEGABTU). This system operates from a minimum generation of 1353 MW to a maximum generation of 3695 MW.

Figure 4. shows the convergence pattern of system incremental cost at system demand of 2400 MW. The expected target of incremental cost, as calculated with the mean distribution method, leads correctly and quickly to convergence. The optimal power dispatch with system demands rising from 2400 MW to 2700 MW is shown in Table 2. Fuel changes occur in Unit 1 and Unit 6 when system demand varies from 2400 MW to 2500 MW, and in Unit 9 when system demand varies from 2600 MW to 2700 MW.

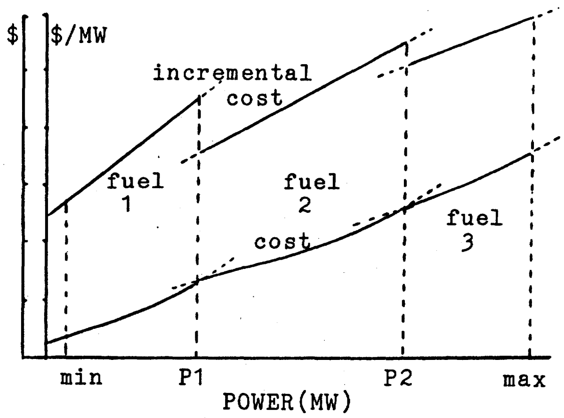

Fig. 1. Hybrid cost and incremental cost function.

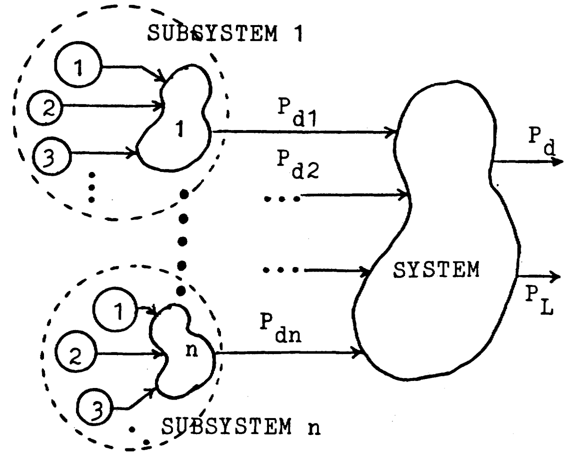

Fig. 2. Hierarchical Structure of Power System.

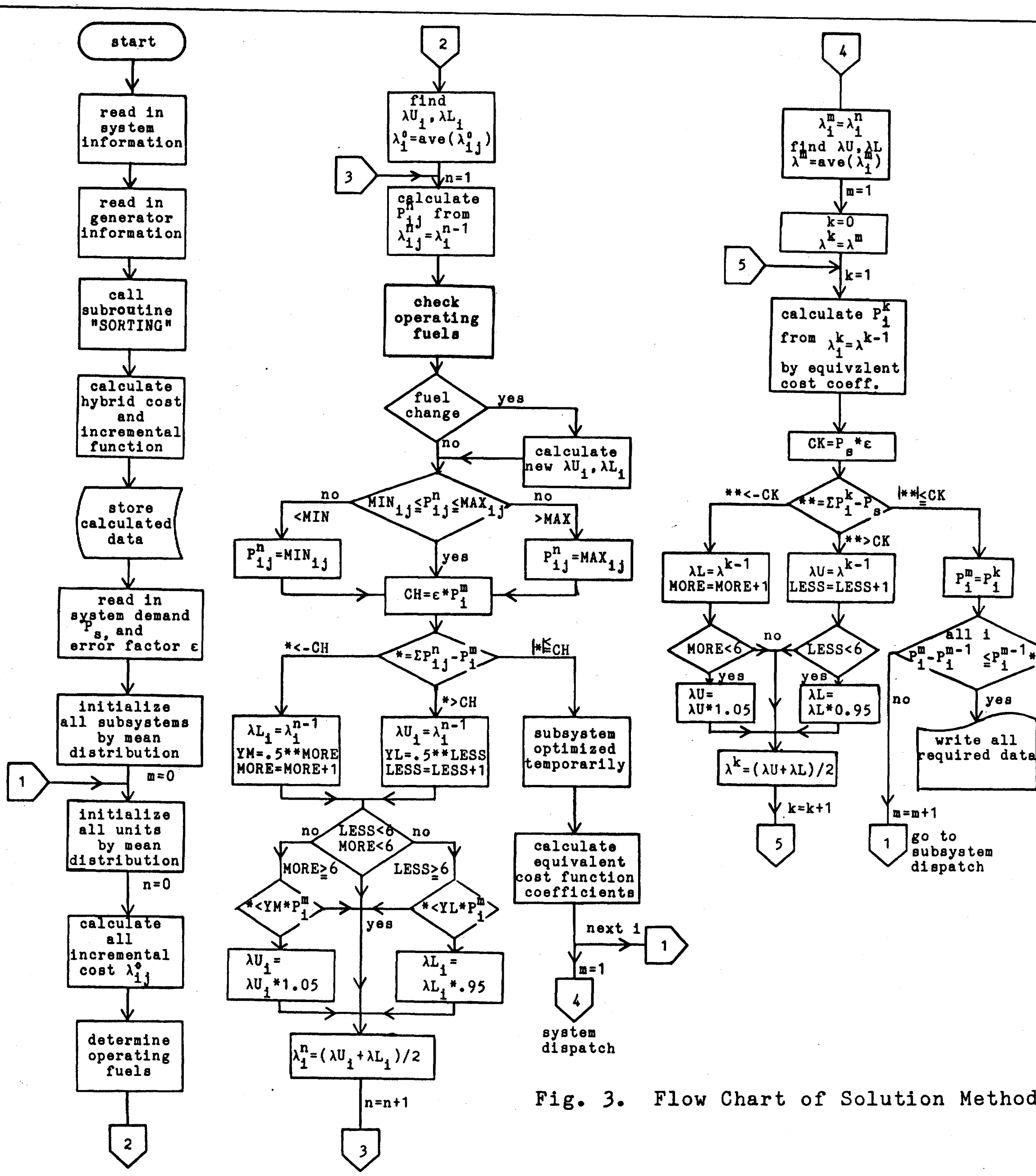

Fig. 3. Flow Chart of Solution Method.

S	U	MIN	F1	P1	F2	P2	F3	MAX	F	A	B	C
1	1	100		196		250		0	1	.2697E2	-.3975E0	.2176E-2
			1		2				2	.2113E2	-.3059E0	.1861E-2
	2	50		114		157		230	1	.1184E3	-.1269E1	.4194E-2
			2		3		1		2	.1865E1	-.3988E-1	.1138E-2
									3	.1365E2	-.1980E0	.1620E-2
	3	200		332		388		500	1	.3979E2	-.3116E0	.1457E-2
			1		3		2		2	-.5914E2	.4864E0	.1176E-4
									3	-.2876E1	.3389E-1	.8035E-3
	4	99		138		200		265	1	.1983E1	-.3114E-1	.1049E-2
			1		2		3		2	.5285E2	-.6348E0	.2758E-2
									3	.2668E3	-.2338E1	.5935E-2
		449						1245				
2	5	190		338		407		490	1	.1392E2	-.8733E-1	.1066E-2
			1		2		3		2	.9976E2	-.5206E0	.1597E-2
									3	-.5399E2	.4462E0	.1498E-3
	6	85		138		200		265	1	.5285E2	-.6348E0	.2758E-2
			2		1		3		2	.1983E1	-.3114E-1	.1049E-2
									3	.2668E3	-.2338E1	.5935E-2
	7	200		331		391		500	1	.1893E2	-.1325E0	.1107E-2
			1		2		3		2	.4377E2	-.2267E0	.1165E-2
									3	-.4335E2	.3559E0	.2454E-3
		475						1255				
3	8	99		138		200		265	1	.1983E1	-.3114E-1	.1049E-2
			1		2		3		2	.5285E2	-.6348E0	.2758E-2
									3	.2668E3	-.2338E1	.5935E-2
	9*	130		213		370		440	1	.8853E2	-.5675E0	.1554E-2
			3		1		3		2	.1530E2	-.4514E-1	.7033E-2
									3	.1423E2	-.1817E-1	.6121E-3
	10	200		362		407		490	1	.1397E2	-.9938E-1	.1102E-2
			1		3		2		2	-.6113E2	.5084E0	.4164E-4
									3	.4671E2	-.2024E0	.1137E-2
		429						1195				
TOTAL		1353						3695				

TABLE 1. System Characteristics of Example
S: Subsystem, U: Unit, F: Fuel,
A, B, C: Cost Coefficients in Equation 1,
Min, P1, P2, Max: Breakpoints in Figure 1,
F1, F2, F3: Operating Fuel Between Breakpoints.
* Unit 9 is a special case.

S	U	2400 MW		2500 MW		2600 MW		2700 MW	
		F	GEN.	F	GEN.	F	GEN.	F	GEN.
1	1	1	193.2	2	206.6	2	216.4	2	218.4
	2	1	204.1	1	206.5	1	210.9	1	211.8
	3	1	259.1	1	265.9	1	278.5	1	281.0
	4	3	234.3	3	236.0	3	239.1	3	239.7
2	5	1	249.0	1	258.2	1	275.4	1	279.0
	6	1	195.5	3	236.0	3	239.1	3	239.7
	7	1	260.1	1	269.0	1	285.6	1	289.0
3	8	3	234.3	3	236.0	3	239.1	3	239.7
	9	1	325.3	1	331.6	1	343.3	3	429.2
	10	1	246.3	1	255.2	1	271.9	1	275.2
		.443452		.463100		.499806		.507248	

TABLE 2. Example/ System Demand Varies From 2400 MW to 2700 MW.
S: Subsystem, U: Unit,
F: Operating Fuel,
GEN.: Unit Generation(MW).

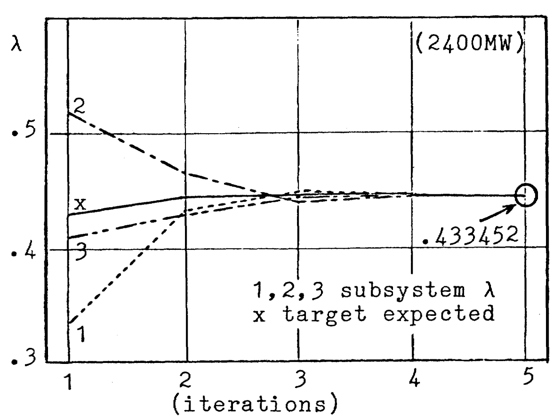

Fig. 4. Convergence Pattern of Example

Computation takes .99 CPU seconds (VAX 11/780) to read in system data and calculate hybrid functions, 0.09 CPU seconds to run system dispatch for a demand of 2400 MW.

DISCUSSION

Because of the discontinuity of the incremental cost curves more than one solution satisfying the equal incremental cost criteria for optimality may result. This situation is characterized in Figure 1 and Equation 2 where an alternative solution may result for some values of λ . For $\lambda = 0.463100$ and system demand equal to 2500 MW, two operating points having the same incremental cost exist. An output of 197.8 MW or 206.6 MW, for Unit 1, as shown in Table 2, result in the same incremental costs. A similar result is found for Unit 6 at outputs of 199.0 MW and 236.0 MW. The optimal solution chosen by the algorithm of this paper requires that Unit 1 operate at 206.6 MW and Unit 6 operate at 236.0 MW. This solution was chosen because the system demand was rising. This choice will minimize the chance of unnecessary switching of fuels, assuming the system demand continues to rise. If the system demand was assumed to be decreasing, a different optimal solution may have resulted. Hence, this algorithm, unlike conventional economic dispatch, requires information concerning the projected increase (decrease) of system demand. This information allows an optimal choice, between two operating points satisfying equal incremental cost criteria, to be made.

A comparison of this method to conventional economic dispatch is difficult since conventional approaches do not account for multiple fuel supplies. A primary purpose of this approach is to provide an on line solution method which avoids unnecessary fuel switching. By limiting the ability to switch fuels (in the solution process), the true exact solution may not result. The authors believe that the benefits of not switching unnecessarily are sufficient to justify possible inaccuracies in solution. To test this hypothesis more fully, it is necessary to develop another algorithm, which would likely not be fast enough for on line application, to determine a true exact solution. This is a difficult task. As an alternative, the authors have presented a practical example which justifies the approach.

CONCLUSION

This research presents a hierarchical method for economically dispatching generation units subject to hybrid or piecewise quadratic operational cost curves. This allows for decentralized determination of appropriate cost functions. Determining cost functions at the plant level is useful for improving accuracy. The efficiency of the method is greatly improved by the mean distribution method which minimizes the time required to converge to a solution. Additionally, the mean distribution also helps to insure convergence to potentially ill-defined problems. The iteration time is short compared with other problems [1]. The overall efficiency of the decentralized computations makes this method attractive.

Based on the results of the example, when system demand varies, the optimal solution for each generation unit does not change abruptly. Additional research is necessary to insure that frequent switching of fuel type (cost curve segments) does not result from the proposed algorithm. From an operational perspective, frequent switching of fuel suppliers is undesirable. The idea of real time application of this solution method is viable especially when an on-line indication of the operating status of each generation unit is available.

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Appendix A

The "mean distribution" method is utilized to distribute the subsystem demand to all generation units by equal percentage of their capacities. If

$$P_{cs} = \sum_i (P_{ci}), \qquad (1)$$

where

P_{cs} = subsystem capacity,

P_{ci} = generation capacity of unit i.

As long as the initial subsystem demand is determined as p(demand), the required generation of each unit will be calculated by

$$P_i = p(\text{demand}) \times \frac{P_{ci}}{P_{cs}}. \qquad (2)$$

Notice that

$$\frac{p(\text{demand})}{P_{cs}} = \frac{P_i}{P_{ci}} = \text{constant}. \qquad (3)$$

Since P_i is related to P_{ci}, larger units will receive a greater initial fraction of the total subsystem demand.

Appendix B

Each segment of the piecewise quadratic cost function for each generation unit is written as

$$COST(P_i) = A_i + B_i \times P_i + C_i \times P_i^2. \qquad (1)$$

The corresponding incremental cost for each segment is calculated as

$$INCS(P_i) = B_i + 2 \times C_i \times P_i. \qquad (2)$$

The coefficients A_i, B_i, C_i are known. P_i is the generating power. For all cases, the C_i coefficients are positive and non-zero numbers. It is assumed that the initial incremental costs of all generation units are determined by the mean distribution method. From this range of initial incremental costs, the greatest value, λ_i(greatest), and the smallest value, λ_i(smallest), is determined. The optimality condition of the subsystem power dispatch requires that all generation units operate at the same incremental cost. The resultant subsystem incremental cost for optimal performance is bounded,

$$\lambda_i(\text{smallest}) \leq \lambda_s \leq \lambda_i(\text{greatest}), \qquad (3)$$

where λ_s is the optimal incremental cost.

This result is verified by first assuming,

$$\lambda_s > \lambda_i(\text{greatest}).$$

If this situation existed, then the total generation of this subsystem would be greater than the subsystem demand. Alternatively, if

$$\lambda_s < \lambda_i(\text{smallest}),$$

the resulting total generation would be less than the subsystem demand. Hence, the subsystem incremental cost must be bounded as shown in Equation 3.