IEEE Transactions on Power Apparatus and Systems, Vol. PAS-104, No. 7, July 1985

PRACTICAL OPTIMIZATION

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Abstract- The recent escalation in fuel costs has resulted in a host of optimization problems which were non-existent when fuel was relatively inexpensive. Coupled with the increased costs have been more difficult operational constraints imposed by fuel vendors. These problems are particularly acute in natural gas fired units which cannot rely on reserve storage of fuel to meet demand. This research addresses the problem of optimal system operation subject to more contemporary operating constraints. The solution algorithm is based on a new optimization algorithm termed "practical optimization".

1.0 INTRODUCTION

The subject of this paper is a new optimization algorithm for power system related problems. In this paper, the algorithm is applied to solve the problem of operating a power system at minimum cost, while satisfying operational constraints resulting from contractual limitations on the fuel supply to the generation units. This research is motivated due to the actual operational problems of a large Southwestern Utility which is heavily dependent on fossil generation units. The algorithm is adaptive in the sense that it will execute faster and faster as more information describing the feasibility space [13-15] of the optimal solution becomes available. It is also adaptive in the sense that it can respond quickly to changes in system operating constraints or objective functions.

Conventional approaches to optimizing power system performance are traditionally separated into two sub-problems. Often, there is the planning problem, usually addressed by a unit commitment algorithm [1-3]. There is also the real time dispatch problem which has a well known solution (economic dispatch) [4-6]. Unfortunately, fuel related constraints vary with both time and system demand. Some research has been done on this problem [7]. Typically, there is a certain amount of fuel which flows into a fuel distribution network, in a specified time period. Multiple units share a single source of fuel. It is the utility's responsibility to assure that the contractual limitations, regarding the flow of fuel into the distribution network, are satisfied. It is also desirable to operate the system at minimum cost.

84 SM 613-6 A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1984 Summer Meeting, Seattle, Washington, July 15 - 20, 1984. Manuscript submitted August 31, 1984; made available for printing May 3, 1984.

For the utility of interest in this paper, natural gas is the major source of fossil fuel. Natural gas contracts limit the amount of fuel available to units within a single plant or within multiple plants. Additionally, it is desirable to take advantage of inexpensive fuel that becomes available, on short notice, on the open market. To provide this flexibility, it is necessary to develop an optimization algorithm which is easily adapted to changing operating conditions and objective functions.

Conventional unit commitment (UC) algorithms are not suitable candidates for modification or expansion to solve fuel related constraints because: 1) UC requires a predicted load demand which is inherently inaccurate. If the actual load is not as predicted, the usefulness of UC is questionable and may cause a violation of a contractual fuel constraint. 2) Typically, UC algorithms do not execute fast enough for real time applications. Therefore, they cannot respond to sudden fuel pressure drops (caused by curtailments from the supplier) or the sudden availability of inexpensive fuel from the spot market.

The optimization algorithm of this paper is termed "practical optimization." It was developed, specifically, to address the problem of optimizing system performance subject to fuel related constraints. Other applications are likely. This method is an unconventional approach to optimization. Additionally, it is well suited to take advantage of feedback information regarding the flow of fuel into the units which is available. Plant computers provide a real time indication of the flow of fuel (gas and/or oil) into generation units. This information is useful for determining how the system is operating with respect to fuel constraints. This feedback information provides necessary information to update fuel constraints if fuel burn rates are outside desirable limits.

The principle behind practical optimization is as follows. First, any fuel related constraints are represented as linear inequality constraints on the control vector as described in Appendix A. Conventional economic dispatch is utilized to determine an optimal operating point, x^d, in real time, where x^d is a vector of MW outputs for each of the generation units. If all fuel related inequality constraints are satisfied (in addition to conventional constraints), the resultant optimal operating point, x^d, is the true optimum. If there is a violation of one or more of the fuel constraints, it is necessary to transform the associated infeasible optimal operating point to feasibility, in real time, in an optimal sense. To accomplish this task, the method of practical optimization is applied.

Some assumptions are presented.

Assumption 1 - It is noted that many cost minimization problems for power systems have objective functions which are relatively flat and continuous [8].

Therefore, many different operating points in the vicinity of the true optimal solution will result in "approximately" the same objective function cost as long as the euclidean distance between any point and the true optimum is sufficiently small.

Assumption 2 - To make the solution to the fuel constrained optimization problem (FCOP) viable, any proposed algorithm should operate with minimal impact on existing power system control center software, yet operate in real time.

Assumption 3 - Any proposed algorithm must be simple to use. It must be available to system operational personnel to aid in decision making. For example, the question of buying or selling power will affect the ability of the system to meet fuel related constraints. Decisions must be made on short notice.

With the above assumptions in mind, some theoretical observations are appropriate. First, it is noted that conventional power system related constraints form a convex polyhedron [9] in n-dimensional (R^n) control space for the control vector x (MW outputs for n - units). This polyhedron set is labelled K^*. The fuel related constraints, represented as linear inequalities, form a convex subset, K ($K \subseteq K^*$). Hence, the optimum determined by economic dispatch, x^d, will also be such that $x^d \in K^*$. However, it is not always true that $x^d \in K$. When $x^d \notin K$, it is necessary to find the practical optimum, x^p. The practical optimum, x^p, is defined as the operating point on the surface of K such that the euclidean distance to the infeasible optimal point x^d is minimum. With the above assumptions in mind, it is further assumed that:

Assumption 4 - The objective function cost at the practical optimum is sufficiently close to the objective function cost at the true optimum.

Note that the practical optimal solution will always be feasible, which is a useful result in itself. Some algorithms, such as Newton-Raphson, cannot guarantee feasibility.

2.0 METHOD

Given the above assumptions and descriptions, the remaining problem is determination of the practical optimum. It is not a trivial matter to determine the point on an n-dimensional convex hull closest to an exterior point. Additionally, it is desirable to accomplish this task in real time. This problem is a cousin to the linear programming problem, however, it is a distant cousin. (The objective function, to minimize the distance between two points, is not a linear relationship.)

Determination of the practical optimum is reduced to two steps: 1) determining the hyperplane [9] which defines the facet (face) closest to the infeasible point x^d, and 2) finding a point on the facet. This problem is still not trivial (see Appendix C). A new algorithm, the ellipsoid algorithm [10-12], provides a solution to this problem. However, several modifications to a basic ellipsoid algorithm (as described in [10-12] and Appendix B) are necessary to achieve fast execution and numerical stability. Basically, the ellipsoid algorithm determines a feasible solution to the system of inequalities,

$$Ax < b \qquad (1)$$

where A is a m x n matrix of constraints, x is the control vector and b is a known constant vector (subject to revision). The algorithm determines a feasible solution by determining (iteratively) shrinking ellipsoids within which a solution is contained. For practical optimization, the matrix A is used to represent both fuel related and conventional constraints. Once the hyperplane closest to the infeasible solution x^d, H^d, is determined, the matrix A is augmented with a hyperplane, H^a (constraint), parallel to H^d and "inside" the set K.

The idea is to force the ellipsoid algorithm to find a solution an epsilon distance from the facet associated with H^d. This goal is accomplished by choosing $\| b^a - b^d \|$ sufficiently small where b^a and b^d indicate the constants associated with hyperplanes H^a and H^d, respectively.

The reader will note that the procedure described above results in determination of a general point essentially on the surface of the facet closest to the infeasible operating point. This does not (in general) result in the closest point (practical optimum). To minimize this error, the original infeasible vector, x^d, is further constrained such that each of its elements may not change by more than a certain percentage. The effect of this is to further limit the feasible solution space, forcing the resulting feasible solution to be sufficiently close to the practical optimum.

3.0 EXAMPLE

For the problem of fuel constrained power system operation, particular care is given to the presentation of an example. Of particular concern is the fact that since this problem has not been solved before, it is difficult to compare the results of this paper to something else. This is particularly true since the method of practical optimization does not result in an exact optimal solution. To have some means of comparison, the method of practical optimization is compared to two different methods. All methods are applied to an actual power system of 14 units. All cost functions, etc. result from actual system data. First, a conventional constrained economic dispatch (CED) is applied. This algorithm addresses the conventional operational limits on the units. Secondly, the cost of operating the system, including fuel related constraints, C^f, is calculated by applying the method of practical optimization (PO). It is apparent that $C^f \geq C^c$ will always result due to the additional fuel constraints, where C^c is the operational cost associated with constrained economic dispatch. Additionally, for a given system demand, it is desirable to determine the practical optimum, x^p, such that $\| C^f - C^c \|$ is as small as possible. If this difference is sufficiently small, the usefulness of the method is verified. Thirdly, once the practical optimum is determined, the cost functions for the generation units are linearized about this operating point. A linear programming algorithm (LP) is applied to determine the minimum. The idea is try to find any (potential) improvements in system cost, as compared to PO. For a system of 14 units (actual system), the three algorithms above are applied. The fuel related constraints have been translated into inequality constraints as described in Appendix A.

In the sample problem, for this paper, the results of a conventional economic dispatch are calculated for 24 different system demand levels. For each CED solution, the results are tested to see if fuel related constraints are violated. If no violations occur, the CED solution is considered the optimal solution. However, for the actual system data, daily system demand, and actual fuel constraints analyzed, 10 times out of 24, a violation of at least 1 fuel constraint occurred. Hence, for these 10 cases, CED resulted in an infeasible solution.

As a typical example of a case of an infeasible CED solution, the case of a system demand of 4352 MW is considered. In total, there are 5 sets of fuel constraints considered. For this example, the only constraint of importance, is the constraint which was violated. This constraint requires that,

$$417 < \sum_{i=1}^{5} \mathrm{SAB}(i) < 1672 \text{ (MW)}.$$

(note: unit names indicate elements of the x vector)

The PO algorithm transforms the infeasible CED solution to a feasible practical optimum as shown in Table 1. For comparison, a Linear Programming (LP) solution, with linearized cost function, is applied to see if further improvements to the practical optimum are possible. (Note: The columns labelled U-BND and L-BND, in Table 1, represent the MW operational limits of the units.

The following are important: 1) The increase in cost of satisfying the fuel constraints with PO, as opposed to ignoring them as in CED, is minimal, 2) an LP algorithm is not capable of determining a less costly system operating point and, 3) the PO algorithm executes quickly (in real time). Also, the cost functions for the units utilized in this example are actual costs. Unfortunately, for confidentiality, these functions have not been included. However, interpretation of the results of practical optimization does not require an explicit descriptions of these cost functions, since they are the same cost functions utilized in conventional economic dispatch.

For an actual application of the method of this paper, the PO algorithm would be included in the power system control software. Additional logic to test for violations of fuel related constraints would also be included. The CED would first attempt to determine a feasible optimal solution. If it fails, the PO algorithm transforms the infeasible solution to the practical optimum, in real time. This approach requires minimal modifications to conventional approaches, yet allows for the flexibility to address a wide variety of operational constraints.

4.0 DISCUSSION

The versatility of the method of practical optimization results primarily from the fact that the method does not specifically address the objective function. The nature of power systems dictate that objective functions are usually "flat" which makes the method of practical optimization a powerful tool for these types of problems. In the limit, as the constraints become severely limiting, the practical optimum will equal the true optimum. Hence, if it could be determined, there exists an exact solution to the FCOP. It is unlikely that the exact solution algorithm to the FCOP would execute in real time or be adaptable to changing objectives or constraints. Hence, the real errors, resulting from changing from the planning mode to the operational mode, would likely exceed the errors associated with practical optimization. Further, practical optimization is significantly more convenient.

Since the PO algorithm need only execute if the results of CED result in a fuel constraint violation, the effects of PO on existing system software are minimized. The present PO algorithm requires about 1 sec of CPU (typically) on an IBM 3083. The code has not been optimized. Hence, for casual or infrequent application of this algorithm (for instance, to solve problems which are insoluble by other means) impact on existing system software CPU requirements is negligible.

OPTIMIZATION :	L-BND		CED	PO	LP		U-BND
SAB1 :	100	≤	230	212.3	191.1	≤	230
NEL3 :	70	≤	70	80.13	72.12	≤	160
SAB4 :	312	≤	512	496.2	446.6	≤	512
SAB5 :	106	≤	475	443.9	399.5	≤	475
SAB3 :	200	≤	317.7	345.9	311.3	≤	438
NEL6 :	247	≤	540	537.3	540	≤	540
WG 5 :	252	≤	436.5	495.9	545.5	≤	550
WG 4 :	250	≤	250	282.8	254.5	≤	500
WG 2 :	93	≤	93	94.02	93	≤	206
LEW1 :	92	≤	260	227.3	245	≤	260
SAB2 :	100	≤	206.3	168.9	185.8	≤	230
WG 3 :	244	≤	296.4	332	365.2	≤	500
NEL4 :	340	≤	425.6	428.2	471	≤	500
LEW2 :	92	≤	239.6	209.4	230.4	≤	260
**********							
TOTAL COST:			1.824E5	1.829E5	1.83E5		

TABLE 1

5.0 CONCLUSIONS

The PO algorithm is extremely effective at minimizing system operating costs subject to fuel related constraints. Also, it is extremely adaptable because of its fast convergence and ability to address a wide variety of objective functions. Applications, other than fuel constraints, such as security constraints and hydro generation constraints, are likely.

Since feedback concerning the fuel supplies is available, the approach is self-correcting, with time. Further, PO satisfies the requirements presented in the form of assumptions in the beginning of the paper.

6.0 REFERENCES

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APPENDIX A

A group of units sharing the same supply of fuel sources is denoted by, G_i, where i usually indicates plant i. c_{ij} is the fraction of generating capacity of unit j as measured with respect to the total capacity of group G_i. N_{ij} is the average efficiency of unit j in G_i. Normally, the contractual limitations dictate minimum and maximum flow rates of fuel (BTU's/Hr) as well as integral constraints on fuel.

Software must allow for the following calculations. First, it must determine the maximum and minimum allowable flow of fuel into G_i. This is measured in BTU's/Hr. This problem is not of great difficulty. Then, the software must convert these numbers to the upper and lower operational limits (MW) for each unit in G_i. This is accomplished by application of the following formula:

$$\max(\min) P_{ij} = c_{ij}N_{ij} \times \max(\min) F_i$$

where F_i is the sum of available fuels for G_i and P_{ij} is the MW output for unit j in G_i.

The formula above is used to obtain the values of the b_k constants described in Inequality 1. When all the G_i's for a particular plant are composed of the exact same set of units, the calculation above is straight forward. Some additional manipulations are necessary when the above condition is not satisfied.

APPENDIX B

The Ellipsoid Algorithm is described in [10-12]. The input to the algorithm is a set of inequalities, $Ax < b$, of size L (see [10]). The output of the algorithm is an n-vector, x, that satisfies $Ax < b$, if a solution exists. The following steps describe the algorithm :

1. Set j=0: $t_0 = 0$: $B = n^2 2^{2L} I$ where I is the identity matrix.

2. j counts the number of iterations. If t_j satisfies $Ax < b$, then return t_j. If $j=16n(n+1)L$, then no solution exists.

3. (Iteration) Choose any inequality in $Ax < b$ that is violated by t_j (denote the corresponding row of A as a^T):

Set:

$$t_{j+1} = t_j - \frac{1}{n+1} \frac{B_j a}{(a^T B_j a)^{.5}}$$

$$B_{j+1} = \frac{n^2}{n^2-1} \left(B_j - \frac{2}{n+1} \frac{(B_j a)(B_j a)^T}{a^T B_j a}\right)$$

$$j = j+1$$

At any point in the algorithm the ellipsoid is described by

$$x: (x-t_j)^T B_j^{-1} (x-t_j) < 1 \quad .$$

A proof of these assertions is found in [11].

The basic algorithm as described above is significantly improved if practical arguments are utilized to limit the size of the initial ellipsoid. This feature was implemented in the algorithm of this paper. The general approach is described in [12]. In this paper, the choice of the initial t, t_0, is the infeasible x^d. The limitations on the changes in the elements of x^d, to determine the practical optimum, are represented by the matrix A.

Procedures for insuring numerical stability are described in [12].

APPENDIX C

This Appendix briefly describes the practical optimization algorithm. The practical optimum is the operating point, x^p, defined as a point such that the Euclidean distance $\| x^p - x^d \|$ is minimum and $x^p \in K$, when x^d is infeasible.

The inequality

$$A^k x < b^k$$

describes the set K^k where the superscript k indicates that the set K^k is subject to change. The faces, F_i, of this set are defined by the intersection of a linear half space, L, and the set K^k

$$F_i = K^k \cap L_i \quad . \qquad (2)$$

The particular, L_i's are determined from the rows of A. If the dimension of $F_i \in R^{n-1}$, the face is termed a facet. A zero dimensional F_i is a vertex.

For practical problems it is sufficient to determine x^p as a point on the face, F_i, such that the distances

$$\{ d_v = \| x^f - x^d \| \; : x^f \in F_i \subseteq K, \; x^d \in K^* \} \qquad (3)$$

$$\{ d_a = \| x^f - x^d \| \; : x^f \in F_j \subseteq K, \; j \neq i, \; x^d \in K^* \} \qquad (3)$$

satisfy the strict inequality $d_v < d_a \quad i \neq j$. Hence, x^f is some point on a face of K which is closest to the infeasible point x^d.

To determine a point, x^p, on the face of K such that (3) is satisfied, the concept of a direction cosine is defined as:

$$\cos u_{12} = \frac{q_1^T \, q_2}{((q_1^T \, q_1)(q_2^T \, q_2))^{0.5}} \qquad (4)$$

for two vectors q_1, $q_2 \in R^n$. The face of K, F_i, closest to an infeasible optimum x^d is found by defining $q_1 = x^d$ and

$$q_2 = q_i \; : \; q_i = a_i^T \qquad (5)$$

where a_i = the i^{th} row of the matrix A of (1). If redundant constraints exist, only those a_i, b_i which are violated by x^d are considered for q_2. The q_i are the normal vectors to a facet (hyperplane) defining K. Since K is convex,

$$\max_{q_2} \; \cos u_{12} \qquad (6)$$

determines the normal vector to the face of K, F_p, which is closest to the point x^d. The practical optimum results from determining any point on F_p by limiting the feasible search space such that the elements of the original infeasible vector are not changed by a specified percentage.

The practical optimum, x^p, is found by augmenting the set of inequalities defined by $Ax < b$ with one more row, equal to the row of A defining the normal to F_p. The equation $a_p x < b_p$ defines the hyperplane of the closest face. The augmented constraint is defined as

$$a_p \, x < b'_{m+1}$$

where b' is chosen such that,

$$x^p \in K: A'x < b', \; (a_p x < b'_{m+1} \cap K) \subseteq K \quad . \qquad (7)$$

Practically, it is not difficult to choose an appropriate b'_{m+1}. If $\| b'_{m+1} - b_v \|$ is chosen sufficiently small, x^p, the practical optimum is specified. (Assuming the solution space has been sufficiently constrained.)

Acknowledgement

The author would like to thank Gulf States Utilities for their support of this research. Also, the helpful comments of the reviewers are appreciated.