An Optimized Procedure for Determining Incremental Heat Rate Characteristics

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Abstract - This paper describes an optimized procedure for producing generator incremental heat rate curves from continually sampled unit performance data. A generalized reduced gradient algorithm is applied to optimally locate "break points" in incremental heat rate curves. The advantages include the ability to automatically take into consideration slow time-varying effects such as unit aging and temperature variations in combustion air and cooling water. The procedure is tested using actual fuel rate data for four generators.

INTRODUCTION

Accurate generator performance characteristics are required in order for economic dispatch and optimal power flow algorithms to work properly. Fuel rate data are usually collected annually or semiannually during "heat run" tests, when units are slowly paced through their operating ranges and steady-state data are gathered at several power points. However, discontinuous data are often obtained due to turbine valving [l], and inaccurate estimates of generator performances limit the accuracy of economic scheduling.

More accurate "up-to-date" information can be obtained by continually monitoring and recording unit performance data during normal operation. The advantage is that, over a period of time, fuel rate data for every integer MW of operating range can be obtained. This has been the subject of work by Viviani, et a1 [2], and more recently by Hillhouse **[3].**

Regardless of how performance data are obtained, optimization algorithms generally require that fuel rate data be in one of the following functional forms: segmented piecewise-quadratic **[4],** cubic, or reduced-cubic with quadratic term omitted *[5].* An example problem with curve fitting is that the slopes are not always positive and increasing (a requirement for most optimization procedures). This subject has received much attention in the literature, and many methods have been devised to improve the chances of successfully curve-fitting the data.

Eliminating the quadratic term (known as the reduced-third order method) often helps, but at the cost of accuracy. An alternative method proposed by Shoults [5] guarantees increasing slopes while using a cubic approximation of the fuel rate curve. Other methods using dynamic programming have also been proposed *[6].* However, the ability to obtain accurate incremental heat rate curves automatically and without human intervention remains a problem.

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The method presented in this paper is designed to produce accurate, piecewise-linear segment approximations of incremental heat rate curves from continuously sampled fuel rate data. accomplished by constructing a simple geometric objective function and using it as input to a nonlinear optimizing computer program. The procedure is tested and evaluated with recorded data for four generators, where **up** to 20 digitally-sampled fuel rate points are averaged for each integer MW of output power. It is designed to work in conjunction with the type of automated unit performance sampling system described in [2-3]. Visual interpretation of unit data or human intervention is not needed. The overall procedure is simple and robust.

SAMPLING TECHNIQUE

A real-time system for sampling unit performance characteristics has been developed and described by Viviani, et al [2], and Hillhouse **[3].** The m most recent fuel rate data samples for every integer MW k are saved in vector \mathbf{F}_k , where

$$
\mathbf{F}_{k} = \left[\mathbf{f}_{k,1}, \mathbf{f}_{k,2}, \cdots, \mathbf{f}_{k,m} \right]^{T} \tag{1}
$$

Viviani uses a value of 20 for m. An average fuel rate for every integer MW k is computed from \mathbf{F}_k as follows:

$$
F_k^{avg} = \frac{1}{m} \sum_{i=1}^{m} f_{k,j} \quad , \tag{2}
$$

where

$$
k = k_{\min}, k_{\min} + 1, k_{\min} + 2, \cdots, k_{\max} ,
$$

and k_{min} and k_{max} are the minimum and maximum rated powers in *MW,* respectively.

Figure *1:* Fuel **Rate** Approximations **for** Generator **1**

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New samples, $f_{k,new}$, are accepted or rejected based on the following criteria:

 $\left| \int_{k, new} - F_k^{avg} \right| > b \sigma_k$, reject, $\left| \int_{k, new} F_k^{avg} \right|$ < b σ_k , accept,

where σ_k^2 is the sample variance of the $f_{k,i}$ points at power **k**, and **b** is a constant. If accepted, the oldest sample $f_{k,1}$ is discarded, the remaining elements of (1) are shifted one location, and $f_{k,m}$ is

replaced with $f_{k,new}$. The F_k^{avg} values for an actual unit are shown as discrete points in Figure 1.

OPTIMIZED PROCEDURE

Overview

The new procedure for finding optimal incremental heat rate curves from sampled data uses nonlinear optimizing computer program GRG2, developed by Lasdon and Warren **[7].** GRG2 is a general purpose program based on the reduced gradient technique that optimizes linear or nonlinear objective functions, subject to equality and/or inequality constraints. The focus of this paper is on how to use GRG2 to solve for incremental heat rate curves, rather than on GRG2 itself.

For introductory purposes, the new procedure can be summarized in the following steps:

- Step 1. Approximate the fuel rate data using a set of J constant-width data-groups, as shown in Figure 2.
- Step 2. Compute average values of fuel rate and power within each data-group, and connect these averages with straight-line segments. Check for increasing slopes of the segments. Merge adjacent data-groups as needed to obtain increasing slopes.
- Step **3.** Formulate the error function illustrated by the shaded area of Figure 3.
- Step 4. Use optimizing program GRG2 to minimize the error function by adjusting the delimiters of the data-groups, as shown by varying stairstep widths in Figure 4.
- Step 5. Compute a stairstep approximation for the incremental heat rate curve from the slopes of the straight-line segments in Figure 3. Connect the mid-points of the stairsteps to obtain the piecewise-incremental heat rate curve shown in Figure 5.

The entire procedure is illustrated in the flowchart shown in Figure 6. Details of each step follow.

Figure **2:** Constant-Width Data-Group Approximation **for** Generator **1**

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Figure **3:** Error Function **for** an Expanded Portion **of** Averaged Sample Data **for** Generator **1**

Figure *5:* Incremental Heat Rate **for** Generator **1**

Figure 6: Flow Chart for Optimized Procedure

Step 1: Initialization

Individual points shown in Figure 2 are the computed F_k^{avg} values from **(2).** An expanded view of these is shown in Figure 3. At the beginning of the procedure, these points are gathered into twenty data-groups, shown as constant MW-width increments on the horizontal axis of Figure 2.

Step 2: Averaging and Merging

For each data-group, average fuel rates and average powers are computed. The height of the stairsteps in Figures **2** and **3** correspond to these average fuel rates. They are represented by the following set of J points:

$$
\left[\overline{\left(\bar{\mathbf{p}}_1, \bar{\mathbf{f}}_1 \right)}, \overline{\left(\bar{\mathbf{p}}_2, \bar{\mathbf{f}}_2 \right)}, \cdots, \overline{\left(\bar{\mathbf{p}}_J, \bar{\mathbf{f}}_J \right)} \right]. \tag{3}
$$

Initially, J equals J_{max} , and the MW delimiters of the stairsteps are

$$
\left[\begin{array}{c}d_1, d_2, \cdots, d_J, d_{J+1}\end{array}\right].
$$

Delimiters d_1 and d_{J+1} correspond to k_{min} and k_{max} , respectively. After computing the averages in (3), the J points are connected with straight-line segments, shown as the upward-sloping lines in Figure 3.

After connecting the average points, there are two possible outcomes: either (a) the slopes of the straight-line segments are increasingly positive, or (b) they are not. The two corresponding courses of action are:

a. For the first pass, the procedure continues with Step 3. Otherwise, the newest delimiters are compared to the previous set. If the largest single change **is** less than 1 MW, then a solution has been reached, and the procedure continues with Step 5. For larger changes, the procedure continues with Step 3, and another complete iteration is performed.

b. If a check of the segment slopes (working in the positive MW direction of the horizontal axis of Figure 3) reveals that a segment slope is less than that of its left-adjacent neighbor, the two corresponding data-groups are merged. This reduces the total number of data-groups by 1, leaving (J - 1) pairs of points in (3). The procedure then returns to the beginning of Step 2. Merging continues until either all the slopes are increasingly positive or until J is reduced to four.

If J is reduced to four, then J_{max} is decremented by one, J is reset to J_{max} , the right-most data-group is deleted, and the procedure is restarted with Step 1 with the remaining J_{max}

data-groups, provided that at least 80% of the original F_{ν}^{avg} values are still included. If fewer than 80% are retained, then a "default" action is taken where the results of the last successful iteration are used, and the procedure continues with Step *5.*

This merging process was developed and refined using the four examples included in the paper and has been shown to work very well.

Step 3: Error Function

The error function to be minimized by GRG2 is represented by the shaded area in Figure 3. It **is** the area between the straight-line segments and the stairsteps. GRG2 adjusts the *Mw* placement of the stairstep delimiters to minimize the total shaded area.

The expression for the shaded area in Figure 3 is shown in the Appendix at the end of the paper and in **[8]** to be

$$
e(\overline{p}, d, s) = \sum_{j=1}^{J-1} \left\{ \left[(\overline{p}_{j+1} - d_{j+1})^2 + (d_{j+1} - \overline{p}_j)^2 \right] \cdot s_j \right\} +
$$

$$
(\overline{p}_1 - d_1)^2 \cdot s_1 + (d_{j+1} - \overline{p}_j)^2 \cdot s_{J-1} , \qquad (4)
$$

where s_i is the slope of the jth segment.

Step 4: Delimiter Adjustment

GRG2 uses a reduced gradient iterative technique to adjust delimiters d_2 through d_1 to minimize $e(\overline{p},d,s)$ subject to

$$
d_j - d_{j\text{-}1} > 0, \text{ for } j = 2, 3, \cdots, J\text{+}1 \ .
$$

The tendency is to move the delimiters closer together in the steepest part of the fuel rate curve, which is usually the right-most portion. New data-groups are then formed.

After successfully completing Step **4,** the solution procedure returns to Step **2.**

Step 5: Approximation of Incremental Heat Rate Curve

A stairstep approximation to the incremental heat rate curve, obtained from the slopes of the line segments shown in Figure 3, is illustrated as the dotted line in Figure *5.* A piecewise-linear approximation to these is obtained by connecting the mid-points of the stairsteps, shown by the solid line. The solid line is extended from the middle of the first and last stairsteps toward the curve boundaries with a very small positive slope, thus holding the incremental heat rate approximately constant in these regions.

RESULTS

The procedure was tested "off-line" with actual sampled data for four generating units. Each unit exhibits different characteristics that are useful in illustrating the capability of the new optimized procedure.

Unit 1

The average sampled data points for unit 1, shown in Figure 1, are very well behaved in that they are tightly grouped and well fitted by upward-sloping cubic and reduced-cubic polynomials. **A** slight step-increase in the sampled points occurs at approximately **240** MW. This enhances the upward slope of the curves in the maximum power region.

The final stairstep approximation for the fuel rate data is shown in Figure **4,** and the final incremental heat rate approximation is shown in Figure *5.* The number of data-groups is reduced from the initial value of **20** to the final value of *6.*

The final piecewise-incremental heat rate approximation is compared in Figure 7 to those produced by traditional cubic and reduced-cubic polynomials. The results are quite similar.

Figure **7:** Incremental Heat Rate for Generator **1**

Unit 2

Unit 2 exhibits widely-scattered averaged data points, as shown in Figure 8. There is a slight step in the data near **450** MW, and the slope decreases slightly at maximum power.

As shown in Figure 8, the number of data-groups is reduced from the initial value of 20 to 7. The merging process continues until fewer than 80% of the average data points are included, thereby invoking the default action described in Step 2. The resulting incremental heat rate approximation is shown in Figure **9.** The sharp increase in incremental heat rate near 450 MW is due to the upward step in the fuel rate data mentioned above.

Unit 3

The sampled fuel rate data for unit 3, shown in Figure 10, is very straight and tightly grouped. Its slope decreases slightly near maximum power. This is responsible for the fact that the stairstep approximation in Figure 10 does not extend through the maximum power region.

The incremental heat rates are shown in Figure 11. There is general agreement among the methods, except in the highest power region, where the slope of the fuel rate curve decreases slightly. In that region, the true incremental heat rate actually decreases. Since decreasing slopes are not permitted in most economic scheduling algorithms, the optimized procedure is designed to replace decreasing slopes with slightly positive slopes near maximum and minimum powers. This is illustrated in the right-most straight-line segment of Figure 11.

Figure **9:** Incremental Heat Rate for Generator **2**

Figure **10:** Fuel Rate for Generator **3**

Figure **13:** Incremental Heat Rate **for** Generator **4**

Unit **4**

Figure **12:** Fuel Rate for Generator **4**

The data for unit **4,** shown in Figure 12, is badly skewed, and the slope decreases dramatically in the maximum power region. Cubic and reduced-cubic approximations produce downward slopes, as shown in Figure **13.** However, the optimized procedure identifies and utilizes the upward-sloping region of the data, as shown by the stairstep approximation in Figure 12. The corresponding incremental heat rate is shown by the piecewise-linear segments in Figure 13. The downward-sloping region, which begins at approximately **400** *MW,* is represented by a line segment with slightly positive slope.

CONCLUSIONS

There are several advantages gained by using the proposed optimized procedure in conjunction with sampled unit fuel rate data. First, realistic characteristics of generators are used to determine the incremental heat rates. Second, the need for annual or semiannual heat run tests is eliminated. Third, the effect of seasonal variations in unit performance is automatically incorporated. Fourth, the "hands-off' procedure does not require visual interpretation. Fifth, a method of handling fuel rate curves that contain regions with decreasing slopes is made viable.

The procedure is successfully tested "off-line" using actual sampled fuel rate data for four generators that exhibit different characteristics. Average solution time for a generator is approximately 20 seconds on a CDC Cyber 170/750 computer. The procedure is shown to work well when compared to conventional cubic and reduced-cubic curve-fitting methods.

ACKNOWLEDGEMENTS

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APPENDIX

The slope of the jth piecewise-segment that connects average points $(\overline{p}_j, \overline{f}_j)$ and $(\overline{p}_{j+1}, \overline{f}_{j+1})$ equals

$$
s_j = \frac{y_{j+1} - f_j}{d_{j+1} - \overline{p}_j} = \frac{\overline{f}_{j+1} - y_{j+1}}{\overline{p}_{j+1} - d_j}
$$

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where y_{j+1} is the intersection of the data-group delimiter and the piecewise-segment. This intersecting value of y for delimiter d_{i+1} is

$$
y_{j+1} = \bar{f}_j + s_j (d_{j+1} - \bar{p}_j) = \bar{f}_{j+1} - s_j (\bar{p}_{j+1} - d_{j+1}).
$$

An error equal to the total area enclosed between the stairstep approximation and the piecewise-segment can be expressed by defining triangles A_{2i} and A_{2i+1} , as shown in Figure A1, as

$$
A_{2j} = \frac{(d_{j+1} - \bar{p}_j) \{ [\bar{f}_j + s_j (d_{j+1} - \bar{p}_j)] - \bar{f}_j \}}{2}
$$

\n
$$
= \frac{s_j}{2} (d_{j+1} - \bar{p}_j)^2,
$$

\n
$$
A_{2j+1} = \frac{(\bar{p}_{j+1} - d_{j+1}) \{ \bar{f}_{j+1} - [\bar{f}_{j+1} - s_j (\bar{p}_{j+1} - d_{j+1})] \}}{2}
$$

\n
$$
= \frac{s_j}{2} (\bar{p}_{j+1} - d_{j+1})^2.
$$

The sum of the two triangles is

$$
A_{2j} + A_{2j+1} = \frac{s_j}{2} \left[\left(\overline{p}_{j+1} - d_{j+1} \right)^2 + \left(d_{j+1} - \overline{p}_j \right)^2 \right],
$$

and the total error for all data groups is given by

$$
A_2 + \cdots + A_{2J-1} = \sum_{j=1}^{J-1} \frac{s_j}{2} \left\{ \left[\left(\overline{p}_{j+1} - d_{j+1} \right)^2 + \left(d_{j+1} - \overline{p}_j \right)^2 \right] \right\}.
$$

The total error function is then written as

$$
e(\overline{p}, d, s) = \sum_{j=1}^{J-1} \left\{ \left[(\overline{p}_{j+1} - d_{j+1})^2 + (d_{j+1} - \overline{p}_j)^2 \right] \cdot s_j \right\} +
$$

$$
(\overline{p}_1 - d_1)^2 \cdot s_1 + (d_{j+1} - \overline{p}_J)^2 \cdot s_{J-1} ,
$$

where the last two terms reflect the error due to the first and last triangles. Delimiters d_1 and d_{J+1} remain fixed and correspond to k_{min} and k_{max} , respectively.

 \sim 1000 cm $^{-1}$ and \sim

Figure Al: Error Function for an Expanded Portion **of** Averaged Sample Data for Generator **1**

BIOGRAPHIES

Antonio H. Noyola, (M, 1986) was born on July 13, 1961, in Mexico City. He received the BS degree in Mechanical/Electrical Engineering at the Instituto Tecnologico y de Estudios Superiores de Monterrey, Mexico, in 1984, and the MSE degree (specializing in Electrical and Computer Engineering) from The University of Texas at Austin in 1987.

During 1984-85, he was a plant engineer for Procter & Gamble de Mexico, Mexico City. In 1985, he began his graduate studies at the U. T. Austin, where he is presently working toward a PhD degree in Electrical and Computer Engineering. He is a Graduate Research Assistant for the Center for Energy Studies at U. T. Austin.

W. Mack Grady, (SM,1983), was bom on January 5, 1950, in Waco, Texas. He received the BSEE degree from The University of Texas at Arlington in 1971 and the MSEE and PhD degrees from Purdue University in 1973 and 1983, respectively.

From 1974 through 1980 he was employed as a system planning engineer at Texas Power & Light Company (now TU Electric), Dallas. After receiving the PhD, he joined The University of Texas at Austin, where he is currently Associate Professor of Electrical and Computer Engineering. His areas of interest include power system analysis and operation, power system harmonics, power quality, and short-term load forecasting.

Dr. Grady is a member of Eta Kappa Nu and Tau Beta Pi. He is the chairman of the IEEE Working Group on Power System Harmonics and a registered professional engineer in Texas.

Gary **L.** Viviani, (SM,1988), was born on June 1, 1955. He received the BSEE, MSEE, and PhD degrees from Purdue University in 1977, 1978, and 1980, respectively. His research interests have encompassed various problems in real-time control of many kinds of large-scale systems, including power plants and power systems. Recently, he devoted a considerable amount of attention to generalized nonlinear periodical systems and devices, including neutral networks.

At present, he is a Principal Engineer at Textron Defense Systems, Wilmington, MA, where he is responsible for developing real-time electro-optical surveillance systems within the Surveillance Systems Directorate. He was previously employed as the Gulf States Utilities Research Professor, where the work for this paper was conducted, and as a research scientist at Du Pont, Wilmington, DE, where he studied nonlinear material properties of polymers.

Dr. Viviani is a member of Eta Kappa Nu and Sigma Xi, and he is a registered professional engineer in Texas.

Discussion

ROBERT K. GREEN, and SHAHRIAR SAHBA, (Central and South West Services, Inc., Dallas, Texas): The authors are to be congratulated on their presentation of a procedure for producing piecewise-linear incremental heat rate curves based on continuously sampled fuel rate data. The material presented is concise and well written. As noted by the authors, the availability of accurate incremental heat rate data is essential for the proper execution of economic dispatch calculation programs.

The economic dispatch computer programs in an energy management system that use piecewiselinear heat rate curves, typically have an upper limit for the number of segments allowed for modelling each generating unit. It would be beneficial to discuss how the procedure described in this paper can be modified to limit the number of heat rate segments to the maximum number of segments allowed in a given energy management system. Similarly, it would be desireable to discuss how the optimality of the incremental heat rate model **is** affected by limiting the maximum number of segments allowed to a predetermined value.

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A. H. NOYOLA, W. M. GRADY, *G.* **L. VIVIANI:** The authors would like to thank the discussers for their helpful comments and questions. The proposed algorithm has been modified accordingly to allow a predetermined number of incremental segments for the generators. This has been accomplished by monitoring the number of segments representing the generator during the iterative procedure. When the number of segments exceeds the predetermined number, the algorithm decrements the number of datagroups by merging the two adjacent groups which have the least change in slope. This procedure is repeated until the desired number of segments is obtained. The new algorithm is shown in Figure **A2,** where modifications are in bold type.

The modified algorithm has been tested using five predetermined segments for the four generators illustrated in the paper. [Figures](#page-7-0) **A3** - **A6** show the corresponding incremental heat rate approximations. Comparing these to Figures 7, 9, 11 and **13** shows significant differences for generator **2** only. This is because the original algorithm failed to find an optimum for generator **2,** and the "default" previous iteration was used. For this case, the modified algorithm appears to provide a better approximation.

The position of optimal break points differs for different numbers of segments. The "optimum" number of segments is not obvious.
Intuitively, using a larger number of segments may result in a more accurate representation, but it may not yield a convex approximation. Based on results using the data for the four generators, it appears that five or six segments provide the best results.

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Figure A2: Modified Flow Chart for Optimized Procedure

Figure A3: Incremental Heat Rate for Generator 1

Figure A5: Incremental Heat Rate for Generator 3

Figure A4: Incremental Heat Rate for Generator 2

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Figure A6: Incremental Heat Rate for Generator 4