On Synthesis and Design of Multi-stable Devices

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ABSTRACT - In light of the growing interest in multi-stable phenomena (multiple stable equilibria), especially with regard to neural networks, phase transitions and autonomous intelligent devices, the need for a means to systematically synthesize (design) desirable devices is apparent. This paper couples physically realizable "building blocks," the elements of which have been provided by L. O. Chua and coauthors in recent works, in a manner consistent and convenient to the needs of the multi-stable device theories developed by Saet and Viviani. The result is a systematic procedure for realization of devices with a variety of desirable and predetermined sets of equilibria.

1. Introduction. Besides the works of Saet and Viviani [1,2], synthesis and design of non-neural network approaches to dynamical systems having multiple stable equilibria have received no direct and focused attention. On occasion, one can locate works with central themes that are related to issues concerning the synthesis or design of a multi-stable device, however, physical construction of an operative device has not been generally considered (except as previously mentioned). A good example of a work that addresses some important issues, but is less than adequate regarding hardware realization, is [3].

In a companion paper, [4], more of the qualitative features of the topology of the phase plane for devices having multiple equilibria, subject to simple parameter variations, are investigated. There, some mention of the formulation of the nonlinear characteristic of the associated Lienard oscillator is provided.The intent of this paper is to provide some insight into how the specific characteristics are "created."

Having already established a reliance on a Lienard (Rayleigh) class oscillator [1,2,4]:

$$\ddot{u} + \mu f(u)\dot{u} + u = 0, \qquad (1)$$

present efforts are focused on systematic procedures for realizing "desirable" nonlinearities, f(u), in order to achieve a pregiven number of stable states and expected structural changes to the topological structure of the phase plane, for variations in parameter μ. Roughly, the resultant qualitative changes to the phase portrait represent a sort of "intelligence," which has not been demonstrated before.

In order to make realization of such devices possible, negative resistance device circuits, which have been documented in [5-7], form the basis for "building blocks" which are then tightly coupled to a mathematical model for a desirable f(u) (or equivalently, F(u), after transformation to the Rayleigh form). Note:

$$F(u) = \int_0^u f(s)\,ds \quad . \qquad (2)$$

2. Method. Without loss of generality, voltage controlled devices with nonlinear characteristic, F(u), are considered, but an equivalent procedure for current controlled devices could also be established.

Relying on some previous results from [5-7] and [1] a voltage controlled type-N [see 5] characteristic can be associated with a circuit of (for example), the topology of Figure 1.

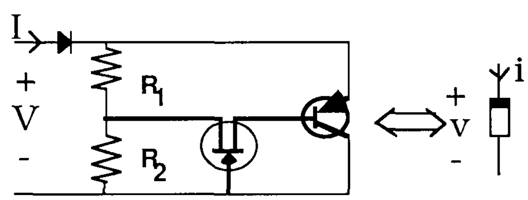

Figure 1

By adjusting various attendant component parameters, such as the resistors R_1 and R_2, the internal characteristics of the JFET or bipolar transistor, and possibly the diode, an I-V characteristic represented by the following sketch in Figure 2.

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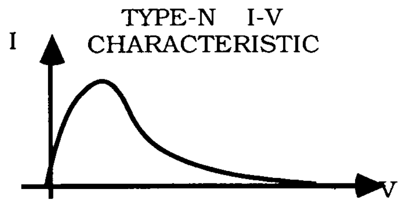

FIGURE 2

By carefully choosing voltage sources which maintain the transistors within normal limits, it is possible to translate the above characteristic along the voltage axis. No doubt, for a particular set of components, device limitations exist, beyond which simple modifications of the I-V characteristic are not possible without changes to the the internal semiconductor properties of the transistors. Such limitations do not, however, prevent fabrication of devices which firmly establish existence of various multi-stable structures.

By simple inspection, one can see that by combining elements of Figure 1 in the manner indicated in Figure 3, an I-V characteristic similar to that in Figure 4 will result. The one shown in Figure 4 is an actual I-V characteristic for a device that was synthesized and simulated with SPICE for particular setsof components in Figure 1 and Figure 3 [1]. The resultant relaxation oscillations are also indicated in Figure 3.

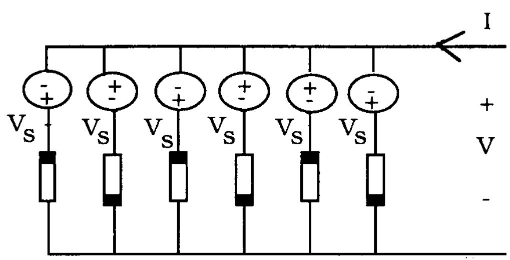

Figure 3

Similar I-V characteristics (received for purposes and with different elemental building blocks) are also presented in [9]. The fact that such desirable characteristics can be received, at this point, is not in question. What follows is an outline of a procedure that is intended to aid in the development of such characteristics.

The type-N device [5,1] has an I-V characteristic whose shape can be thought of as resembling an "un-normalized" gaussian probability density function (p.d.f). By abusing the concepts of mean and variance in a obvious manner, it is constructive to think in terms of a function:

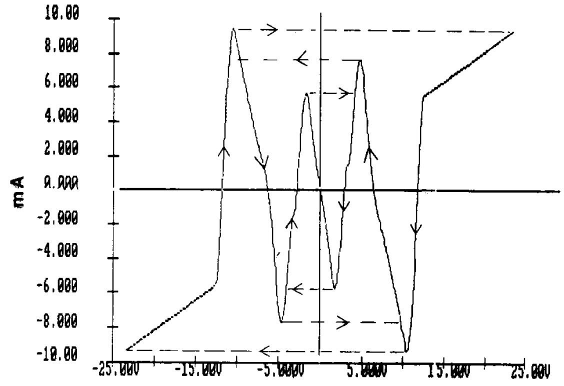

Figure 4

$$A_i \exp\left(\frac{-(x - M_i)^2}{\sigma_i^2}\right) = \Phi_i(x)$$

$A_i \neq 0$
M_i = "mean" (corresponding to unimodal extremum)
σ_i^2 = "variance"

With conventional statistical reasoning, it is easy to show that the majority fo the area "under the hump" is contained within the interval $[M - \sigma, M + \sigma]$. As a rough engineering approximation, the I-V characteristic associated with a type-N device can be approximated by a suitably chosen Φ_k.

In order to obtain nonlinearities which will result in multi-stable operation, which have been previously described and synthesized in [1], the mathematical approximation to the I-V characteristic can be established with a function:

$$\Psi(x) = \sum_{i=-m}^{m} \Phi_i + B x^n , \qquad (3)$$

$$B > 0$$

$n \in \{1, 3, 5, ..\}$ (the odd integers) and certain further conditions exist for the set $\{A_i, \sigma_i^2, M_i\}$ of parameters. The second term in (3) is necessary to insure the stability of the desired oscillations.

For symmetry considerations, $m \in \{2, 4, 6, ...\}$ (the set of even integers) and

$$\sigma_{-k}^2 = \sigma_k^2 \quad , \quad \sigma^2 > 0$$
$$A_{-k} = - A_k$$
$$M_{-k} = - M_k \text{ and } M_{-k} < 0, M_k > 0.$$

With m = 6, B = 1, and n = 1, in (3), and by imposing the conditions above for suitably chosen A_k's, M_k's and s_k^2's, an I-V characteristic that resembles the one of Figure 4 will result . If one relies on Equation (3) as a conceptual aid for developing I-V characteristics, like the one in Figure 4, the next problem (which will not be discussed further here) is to associate the device characteristics of the elements of Figure 1 with the set of parameters $\{A_k, \sigma_k^2, M_k\}$.

When μ is small enough, the associated bifurcating circles whose radii, R, follow from the solution to:

$$\Pi(R) := \int_0^{2\pi} f[R \cos t] \sin^2 t \, dt = 0 \qquad (4)$$

provide the the quasilinear solutions . By varying F(v) interactively on a computer, through parameter and device characteristic variations, and by simultaneously evaluating the roots to (4), the desired number of solutions to (1), for all values of μ, and for desired phase portraits, can be determined.

The above procedures have some inherent approximations and simplifications. For example, it is clear that almost no physically constructed I-V characteristic can be equal to the one indicated by (3). Often, there are asymmetries which are described by terms such as "skewness" or "tails" which are often present. Fortuitously, due to the symmetries characteristic of the proposed procedure, cancelation of undesirable asymmetries usually occurs.

Verifying that a particular nonlinearity will guarantee an oscillation(s) can be a formidable task and will be addressed through numerical simulations, in this paper. A mathematical justification of the described procedure could be developed out of an application of the ideas presented in [3].

3. Numerical Simulations. The following realizations of F_j (Equation (2)), providing phase portraits for "small" and "large" μ are shown in Figures 5 and 6. The functions F_j ,are determined as the following:

$$F_j(X) = \sum_k A_{jk} \exp((X - M_{jk})^2 / \sigma^2_{jk}) + X^3 \quad \text{for } j = (1, 2)$$

j = 1						
k =	1	2	3	4	5	6
A_{jk}	2.20	-1.00	1.00	-2.20		
M_{jk}	-0.90	-0.30	0.30	0.90		
σ^2_{jk}	0.005	0.25	0.25	0.005		
j = 2						
A_{jk}	2.0	-1.50	1.00	-1.0	1.50	-2.0
M_{jk}	-0.80	-0.70	-0.30	0.30	0.70	0.80
σ^2_{jk}	0.001	0.0001	0.5	0.25	0.0001	0.001

Table 1

After standard transformations, (1) can be written in an equivalent form for integral curves:

$$\frac{dy}{dx} = \frac{-x}{\mu^2[y - F(x)]} \qquad (5)$$

where F is defined in (2).

Figure 5. All equilibria and limit cycles are of the hyperbolic type. The symbols UE, SE, SLC, and ULC indicate unstable equilibrium, stable equilibrium, stable limit cycle, and unstable limit cycle, respectively. Numerical phase portraits, dx/dt vs x, for (5) with the characteristic F_2 of Table 1. a) at μ = 0.3, b) at bifurcational value of μ = 2.2, c) at μ = 5.

Figure 6. Numerical phase portraits, dx/dt vs x, for (5) with the characteristic F_1 of Table 1. a) at μ = 0.01, b) at bifurcational value of μ = 1.4832, c) at μ = 4.0, d) characteristic $F_1(x)$ vs x with indicated relaxation curves.

In both cases the qualitative nature of the phase portraits changes for variations in μ. Various numbers of equilibria are received, depending on μ. For further clarification on how to determine the indicated relaxation oscillations, the reader is referred to [8,1,4]

4. Concluding remarks. The indicated procedures for designing circuits with multiple equilibria have successfully been applied for hardware synthesis [1], and have also been verified by numerical simulations.

A cumbersome, but rigorous, mathematical basis for such an approach could be established by applying concepts developed in [3]. The proposed procedure is also verified to be effective for designing circuits requiring qualitative variations to the phase plane structure of the dynamical system of (1).

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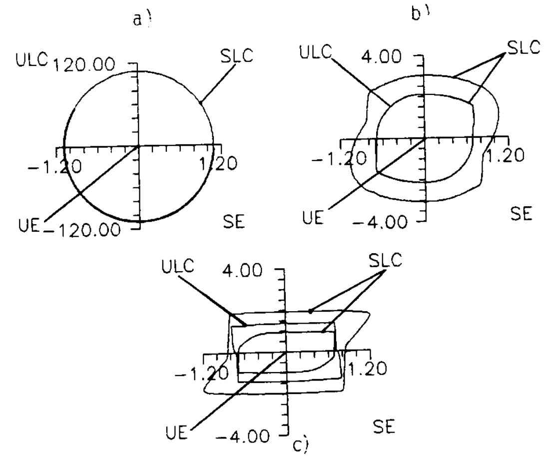

Figure 5

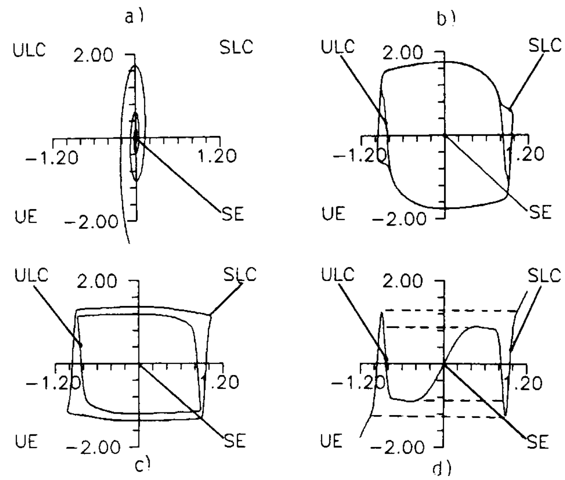

Figure 6