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Generation Scheduling with Integral Constraints on Fuel Supplies

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SUMMARY

Fuel supplies for power system generation units have become more constrained as fossil fuels (especially oil and natural gas) have become more scarce. This paper addresses the issue of integral constraints on fuel supplies which limit the total available fuel energy for a specified time horizon. A comparison with existing methods of optimization is made.

1. INTRODUCTION

The recent scarcity of fossil fuels has led to previously non-existent power system operational problems. The majority of these new problems stem from constraints on quantities of fuel available, especially those imposed by suppliers of natural gas. For a given time horizon of interest, present operational limitations for some utilities require that the total fuel energy, in addition to the instantaneous flow rate of fuel, be constrained to equal, or fall between, definite limits. These constraints greatly complicate the operational strategies for utilities which rely on natural gas as a fuel. Additionally, as fossil fuels become more scarce, and utilities are forced to buy more of their fuels on the spot market, these new constraints are likely to become more common.

An algorithm is presented to address the problem of integral constraints which limit the total available quantity of fuel available to a generation unit in a specified time interval. Previous approaches to power system operational problems do not address integral constraints on fuel energy. (Fuel energy is defined as the total available BTUs of a particular fuel. Usually a time interval over which this fuel is consumed is also specified.) Further, because of the implied time dependence of integral constraints, time-invariant power system optimization methods, such as economic dispatch, are not applicable to this more contemporary problem.

Numerous authors have addressed problems similar to those presented here. In each case, there are differences which render past analyses inapplicable to the specific problem of integral constraints on supplies of fuel. For example, several unit commitment approaches to related problems have been proposed [1 - 4]. These papers, when they have addressed integral constraints, have focused on those related to water supplies. Additional references to this sort include refs. 5 and 6. Recently, others have begun to focus more on fuel-related constraints. References 7 and 8 summarize activities related to the topic of this paper.

2. PROBLEM FORMULATION

First, the relationship between the shape of the system load demand curve and feasible solutions to the problem of integral constraints on fuel are examined. It is the shape of the daily system demand curve which is the source of any improvements on the base load strategy of satisfying the integral constraints. This proposition can easily be proved by a counter-example. Assume the daily load demand versus hours is a constant function. By employing economic dispatch theory at each time increment, it is evident that the optimal strategy for loading unconstrained units will be a constant function. Assume the remaining capacity requirement is fulfilled by the energy constrained units. Now, assume that the generation schedule for an energy constrained unit is made to vary while still satisfying the energy constraint. Assuming two energy constrained units do not have offsetting influences, the generation level of the units which are not energy constrained

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must vary to satisfy the power flow constraint of economic dispatch [9]. By contrast, the optimal dispatch for energy constrained units for constant system demand is also a constant. Hence, only as a result of a time varying system demand is it possible to have a time varying optimal strategy for an energy constrained unit. Further, by virtue of the mean value theorem, a base load strategy will always provide a feasible solution for a periodic system demand and an energy constrained unit (assuming a single unit cannot generate in excess of the system demand).

This paper addresses an important subset of the operational problems associated with minimizing costs subject to integral constraints on the fuel supplies. A feasible operational strategy for an energy constrained unit can always be realized by a base load strategy. This is an obvious result, assuming that the unit is available throughout the time horizon of interest. Additionally, it is assumed that energy constraints can be represented in terms of individual units in a plant. One method of accomplishing this goal is to give priorities to each of the units in a manner similar to the priority of operation in the unit commitment algorithm. The most efficient unit in a plant would be allowed to burn the most fuel. Conversely, the least efficient unit in a plant would be the first unit to have a limited fuel supply in the event that there is sufficient fuel for all but one unit in the plant. By applying this priority ranking to each plant, fuel energy constraints are represented on an individual unit basis. With this assumption, the optimal schedule for a fuel limited unit is found by optimal determination of excursions above and below a known base load strategy. The integral of the total generation for the fuel constrained unit over the time horizon of interest must equal the total MWh of its energy constraint. (Note: if a unit is varying around a nominal base load strategy, there is not a one-to-one correspondence between the total BTUs and the MWh. The proposed method of solution will constrain the MWh for a unit. This will result in inexactly satisfying integral constraints on the total BTUs of available fuel. These errors are expected to be small.)

It was previously argued that reductions in cost below a base load strategy result from the shape of the system demand curve. Based on the shape of this curve, an optimal strategy is determined. For an energy constrained unit, the system demand curve is normalized by the MWh constraint on the energy constrained unit. This normalized curve of normalized unit demand, $q(t)$, is interpreted as the amount of energy that a particular unit is responsible for producing. The energy constrained unit would produce the same amount of energy over the time horizon of interest by either generating at a base load value, P_0, or allowing unit generation, $g(t) = q(t)$ (since $q(t)$ is normalized). These situations are the extremes of the feasible strategy space for the energy constrained unit. Alternatively, the unit generation may follow a path in between these extremes as long as it continues to satisfy the energy constraint. Only one of these strategies results in optimal cost.

The following definitions are appropriate at this time: $g(t)$ = normalized generation, $q(t)$ = normalized system demand, P_0 = base load MW output to satisfy MWh constraint.

So far, the problem formulation has been expressed from the reference frame of a particular energy constrained unit. A reverse perspective is now pursued which will result in an easier formulation. The problem can be equivalently stated as one of producing a certain amount of energy while simultaneously satisfying a normalized demand $q(t)$. A base load strategy requires that the 'hills' and 'valleys' of the demand curve be followed by other units in the system. If a composite generation cost curve for all units except the energy constrained unit is formed, a system perspective is possible.

Consider a normalized system generator responsible for producing an amount of energy equal to the amount of energy of the energy constrained unit. The cost of system generation is found by forming a composite curve of all available generation, excluding energy constrained units, and then normalizing the curve. From the system perspective, the energy constrained unit, operating at a base load strategy, is viewed as a zero cost unit, except if the unit is cycled above (or below) its base load, P_0, value. By virtue of the energy constraint on the unit, cycling above the base load value requires an offsetting cycling below the base load value. A cost associated with the cycling of the unit is determined by integrating the unit generation cost function above and below the nominal value and finding the average value. From the system perspective, the problem is one of determining whether it is better for the normalized system demand to be carried by system generation or to utilize the energy constrained unit to satisfy the demand subject to the energy constraints on the energy constrained unit. The optimal strategy, from the system perspective, will determine the amount of cycling of the energy constrained unit in order to produce a certain amount of energy at minimum cost. The value of the cycling will be added to (subtracted from) the nominal base load strategy for the energy constrained unit to determine the optimal strategy.

In light of the system perspective of the problem, the system cost of normalized generation is represented by

$$\mathrm{GC} = \int_0^T (0.5a_2 g^2 + a_1 g + a_0)\, \mathrm{d}t \tag{1}$$

Additionally, the cost coefficients are calculated based on normalization of all available unit costs throughout the system (see ref. 10, pp. 126 - 129).

Let $u(t)$ be the amount of cycling of the energy constrained unit below the nominal base load strategy, and $v(t)$ be the amount of cycling of the energy constrained unit above the base load strategy. Then the following equation regarding normalized generation must be satisfied at all times:

$$g(t) + v(t) = q(t) + u(t) \tag{2}$$

A state variable, $x(t)$, is formulated to track the rate of change of cycling of the energy constrained unit as

$$\dot{x}(t) = u(t) - v(t) \tag{3}$$

To satisfy the energy constraint,

$$\int_0^T [u(t) - v(t)]\, \mathrm{d}t = 0 \tag{4}$$

Further, it is required that $u(t) > 0$ only when $g(t) < P_0$ and $v(t) > 0$ only when $g(t) > P_0$. Since it is undesirable to have both $u(t)$ and $v(t) > 0$, the following is always true:

$$u(t)\, v(t) = 0 \tag{5}$$

For simplicity, the normalized demand function is assumed to be periodic. This is unrestrictive and it is easy to verify that $x(t) = 0$ (for some time in the time horizon of interest).

The cost of cycling the energy constrained unit will be measured by the following expression:

$$\mathrm{CC} = \int_0^T (b_1 x + b_0)\, \mathrm{d}t \tag{6}$$

Calculation of b_1 will be based on integration of the cost curve associated with the energy constrained unit. Finally, the following must also hold:

$$v(t) > 0, \qquad u(t) > 0, \qquad x(t) > 0 \tag{7}$$

The total cost, J, is expressed as

$$J = \int_0^T [0.5a_2(q + u - v)^2 + a_1(q + u - v) + a_0 + b_1 x + b_0]\, \mathrm{d}t \tag{8}$$

It should be noted that the proposed problem formulation is of the form of a conventional optimal control problem. Existing theory is readily applicable [11]. The Hamiltonian is formulated as follows:

$$\begin{aligned} H(t) &= 0.5a_2(q + u - v)^2 \\ &a_1(q + u - v) + a_0 + b_1 x(t) + b_0 \\ &= [\lambda(t) - \mu(t) + \nu][u(t) - v(t)] \end{aligned} \tag{9}$$

The Lagrangian multipliers $\lambda(t)$, $\mu(t)$ and ν account for the state equation, the state variable inequality constraint ($x(t) > 0$), and the energy constraint, respectively.

The optimal trajectory must satisfy the following:

$$\begin{aligned} &u(t)\, H_u = 0; \quad u(t) > 0, \quad H_u > 0 \\ &v(t)\, H_v = 0; \quad v(t) > 0, \quad H_v > 0 \end{aligned} \tag{10}$$

by virtue of the maximum principle of optimal control [12]. Additionally,

$$\begin{aligned} &\mu(t)\, x(t) = 0; \quad \mu(t) > 0, \quad x(t) > 0 \\ &\dot{x}(t) = u(t) - v(t) \quad \text{and} \quad x(t) = 0 \end{aligned} \tag{11}$$

On the boundary

$$\lambda(t) = -H_x\ \lambda(0) = \lambda(T) = 0 \tag{12}$$

Note that $\lambda(t)$ must be discontinuous to account for the periodic nature of $x(t)$.

Also,

$$\int_0^T [u(t) - v(t)]\,dt = 0 \qquad (13)$$

Noting that

$$-H_x = \text{constant} \qquad (14)$$

$\lambda(t) = b(k - t)$ where k is chosen to account for the discontinuities resulting from the periodicity of $x(t)$.

The problem reduces to determining the optimal generation $g(t)$. This is found by satisfying the optimality conditions above.

3. SOLUTION

To determine the optimal strategy, the conditions for $v(t) > 0$ are first determined. $v(t) > 0$ can only hold when $g(t) > P_0$.

Assume $g(t) > P_0$, then

$$H_v = -a_2(q + u - v) - [\lambda(t) + \nu + a_1] \qquad (15)$$

assuming that $\mu(t) = 0$ when an optimal strategy is pursued with $x(t) > 0$.

Proving by contradiction [11], assume that

$$v(t) > 0 \quad \text{if} \quad g(t) > K_1 + [b_1(t - K_2)]/a_2$$

If we let $v(t) = 0$ such that

$$q(t) > K_1 + [b_1(t - K_2)]/a_2$$

then

$$H_v = -a_2\{q - K_1 - [b_1(t - K_2)]\}/a_2 - a_2 u(t) < 0$$

which is a contradiction to the optimality conditions requiring that eqn. (10) > 0.

Now, assume that $v(t) > 0$, which requires that $H_v = 0$. Then

$$g(t) = q(t) + u(t) = K_1 + [b_1(t - K_2)]/a_2 \qquad (16)$$

where

$$K_1 = -(\nu + a_1)/a_2$$

and K_2 is an appropriate entry point which assures that $x(t) \geq 0$. By a similar argument, it can be shown that $u(t) > 0$ when

$$q(t) \leq K_1 + [b_1(t - K_2)]/a_2$$

These optimality conditions are interpreted as follows.

(1) Draw a line over the normalized demand function with slope equal to b_1/a_2 and intercept K_1. The variable ν is chosen to insure that the area defined by $[q(t) \geq g(t)]$ .AND. $[q(t) \geq P_0]$ is equal to the area defined by $[q(t) \leq (g(t)]$ .AND. $[q(t) \leq P_0]$.

(2) The optimal system normalized generation $g(t)$ is found by following $q(t) = g(t)$ except along the areas defined in (1) where $g(t) = K_1 + [b_1(t - K_2)]/a_2$.

(3) The cycling of the energy constrained unit is found by evaluating the values of $u(t)$ and $v(t)$ determined above. The generation schedule, $z(t)$, for the energy constrained unit is given by

$$z(t) = P_0 + v(t) - u(t) \qquad (17)$$

over the time horizon of interest.

4. EXAMPLE

The actual system demand curve (Fig. 1) has been normalized for an assumed energy constraint of 1725 MWh (23 increments of 75 MW). This translates to a P_0 of 75 MW. The slope b_1/a_1 is assumed to be 0.33, while a_2, a_1 and a_0 are actual values for a typical unit, which would reflect the total system normalized cost. By applying the above theory, the optimal cycling of the assumed energy constrained unit is found by analyzing the effects of the diagonal line, chosen such that the energy constraints are satisfied. The actual generation for the energy constrained unit is indicated as z in Fig. 1. (Note that a time horizon of 48 hours is shown. Actually, a 24 hour time horizon is considered. The initial time starts where the linear diagonal line crosses the normalized demand $q(t)$. The final time is 24 plus the initial time.)

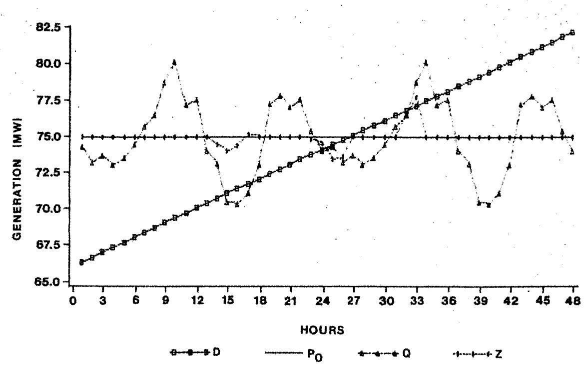

Fig. 1. System demand curve.

5. RESULTS

By studying Fig. 1, it is apparent that if $b_1 = 0$ (no cycling cost) the optimal strategy would be to allow the energy constrained unit to follow the load. For $b_1 > 0$ (but not too big), a line as shown in the Figure is determined. For b_1 much greater than 0, it is possible not to have a solution, which means that the cycling cost of running the energy constrained unit is much greater than the cost of providing generation elsewhere in the system.

6. IMPLEMENTATION

To make the approach of addressing fuel constraints described in this paper usable, it is necessary to describe how to integrate this algorithm into existing software. To begin with, a unit commitment (UC) [1] is run to determine a gross schedule:

(1) Fuel consumptions are determined and compared with contractual limitations.

(2) Based on the tabulated fuel flows, plants where fuel constraints apply are identified. If no contractual violations on integral constraints occur, no further action is necessary.

(3) For plants which burn above or below a contractually defined fuel energy, the algorithm of this paper is applied. All on-line units within a plant, except one, are kept at the output level determined by the UC algorithm, at fuel constrained plants. The remaining unit, at a fuel constrained plant, is initially base loaded to satisfy the fuel constraints at the plant. The coefficients a_0, a_1 and a_2 are determined by utilizing all remaining units in the system as not subject to integral constraints on fuel. The base load strategy above is optimized with the algorithm of this paper.

(4) Once optimal generation schedules for all energy constrained units are determined, their output levels are fixed, and subtracted from the total system demand.

(5) A UC is again run to satisfy the remaining system demand with the remaining unconstrained units.

The term 'quasi' will be used to indicate an algorithm similar to an existing algorithm. 'Quasi' will preface a term which describes existing software which must be modified to achieve the goal of solving the integral constraints on fuel supplies problem. Note: a unit commitment algorithm is assumed to determine both which units to operate and at what output level for economic operation. This assumption is based on the typical capabilities of present unit commitment algorithms.

The elements of the solution algorithm are as follows:

(1) Run a quasi unit commitment (UC) which at first does not consider the effects of the integral constraints. The input to this UC will include the predicted system demand. Further, the quasi UC must optimize performance over the desired time horizon of interest. The quasi UC algorithm will interpret its own output schedule to determine if any of the energy constrained units are in violation of their respective energy constraints. If there are no violations, the operational strategies are unchanged from historical practices.

(2) For plants which are in violation of their allowable fuel energies in the prescribed time interval, two questions must be resolved. (Note: fuel constraints are typically on a per plant basis. Several generation units share the same source of fuel.) First, it is necessary to determine by what amount the fuel limitations have been violated. Second, it is necessary to constrain the operation of individual units, in an optimal sense, in order to avoid violating the fuel constraints at a plant. Determining the magnitude of the fuel energy violation is achieved by reference to a table containing a list of all energy constraints and then calculating the fuel consumptions over the desired time horizon and subtracting. Only two situations can result: either too much or too little fuel is burned. For both situations, the priority start-up list (this is an internal array in most dynamic programming based unit commitment algorithms, and is a rank ordering of all the units in the system in order of their individual average unit efficiencies) will be used to resolve plant-wide fuel constraints to individual units. If too much fuel is burned, reference to the priority list will reveal the least efficient unit at the plant in question. While holding the schedule of the other units at the plant unchanged from the quasi UC results, a base load sched-

ule for the least efficient unit is determined to just meet the energy constraints for the time horizon of interest. Similarly, if too little fuel is burned, a base load strategy of the most efficient unit is determined to just meet the constraint while holding all other constrained units at the fuel constrained plant at their prescribed output levels throughout the time horizon of interest.

(3) Base loaded energy constrained units will determine their optimal schedule by application of the algorithm described in this paper. The necessary system generation cost coefficients are determined by utilizing the cost curves for all units in the system which are not involved in energy constrained conditions. (The assumption is made that not all units in the system will be subject to fuel constraints.) This composite cost curve is readily established by well-known techniques (see, for example, ref. 10).

(4) Once the schedules for all the energy constrained units (quasi base loaded and those with fixed schedules) are determined, they are fixed in the quasi UC algorithm. This quasi UC again determines an optimal schedule with the fixed schedule of the energy constrained units as input.

7. CONCLUSION

This paper has presented a viable algorithm for addressing the problem of optimizing system costs subject to integral constraints on fuel supplies. This is likely to be a growing problem as long as utilities continue to burn fossil fuels (oil and natural gas). A suggested approach for incorporating this solution algorithm with existing software has been provided.

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