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New type of multi-state detector for adaptive control: an application to network switching

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A new type of detecting device with many different states is proposed on the basis of a non-linear oscillator with several stable limit cycles. The device can be used as an intelligent switch. A simple example of possible applications is presented, while numerous adaptive control applications exist.

1. Introduction

In a previous work, we introduced an equation termed the 'conservative multi-stable equation' (CME) and synthesized it in the form of a compact electronic device operating as a multi-state oscillator (Saet and Viviani 1984). The CME was analysed numerically and experimentally to obtain more complete information regarding its dynamics. Thus it is analytically and electronically proven that the CME provides a very reliable basis for synthesis of a multi-state oscillator.

The set of periodical solutions of the CME are indicated as $x_k(t)$ for $k = 1, 2, \dots, N$, where N is an integer. Each state is associated with a different periodical regime, which when electronically synthesized can be used to sense different external stimuli. Perhaps more importantly, different discrete responses to different continuous inputs can occur. A device of this type is termed the 'multi-state detector' (MSD), and is the subject of the present work. The block diagram in Fig. 1 indicates the essential elements of the MSD. This device has properties which are not attainable by computerized approaches since its use for decision-making is not limited by the cycle time of the central processing unit, as in digital computer-based sensors.

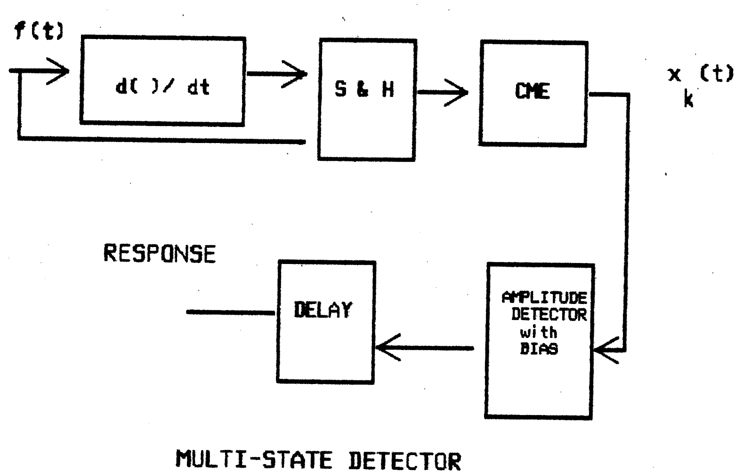

Figure 1. Multi-state detector.

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2. Phase plane structure of the CME equation

As in our earlier paper, the CME is defined as

$$\begin{gathered} \ddot{x} + \mu L(x; \dot{x})\dot{x} + \omega^2 x = 0 \\ L(x; \dot{x}) \equiv \prod_{k=1}^{N} (z^2 - E_k^2) \\ z \equiv \dot{x}^2 + \omega^2 x^2 \end{gathered} \tag{1}$$

This system has solutions which correspond to ellipses

$$\dot{x}^2 + \omega^2 x^2 = E_i \quad E_1 < E_2 < \cdots < E_N$$

in the phase plane $(x, \dot{x})$. Making use of $V(x, \dot{x}) \equiv \dot{x}^2 + \omega^2 x^2$ as a Lyapunov-like function, one can state that the annular domains between cycles are the invariant sets. The outer cycle, corresponding to the periodical solution $x_N(t) = E_N \sin(\omega t + \Phi)$, is always attracting, regardless of the value of N. Alternate cycles are stable, starting from the cycle with number N. Thus, if N is odd (even) then each cycle with odd (even) number is stable (unstable). Since the periodical solutions do not depend on the value of μ, the absolute value of the negative Lyapunov characteristic number of the limit cycle can be made as large as desirable. In practical terms, this means that the desirable small time of convergence after perturbation, to the proper attracting limit cycle, can be achieved for an appropriately chosen μ. This may be a significant advantage of the proposed detector. Characteristics of the time for convergence, depending on the value of μ and the consecutive number of the limit cycle and other details, are given in Saet and Viviani (1984). For example, at $\mu = 2$, the time for convergence is less than $(2\pi/\omega) \times 10^{-1}$.

3. Example of application

In this work, the concept of an MSD device is presented for power system relaying. Thus, it is desirable to produce different responses for conditions constituting different degrees of potential hazard.

The proposed device in Fig. 1 operates under the principle that when initial conditions, resulting from the sample and hold (S&H) block, are applied to the CME block, the output of the CME will be one of the stationary periodic solutions, $x_k(t)$. Hence, the samples produced by the S&H block consist of a series of impulses of varying magnitude separated by time intervals of length T. (The duration of the impulse is neglected.) Then, for a particular time-varying actuation signal $f(t)$, and its derivative $f'(t)$, the set of initial conditions, $X_0 = \{x_0, \dot{x}_0\}$, applied to the CME is composed of $\{f(kT), f'(kT)\}$, $k = 1, 2, \ldots$. Here, T is chosen to be significantly larger than the time to converge to a stable limit cycle, and therefore this time of convergence can be neglected.

Retaining the originally defined numeration of cycles, we define the set A_k to be the domain of attraction for the kth limit cycle, when this cycle is stable. Thus

$$A_k = D_{\text{in}}^{(k)} \cup D_{\text{out}}^{(k)}$$

where $D_{\text{in}}^{(k)}$ and $D_{\text{out}}^{(k)}$ are the sets of points which are attracted to the limit cycle, lying in the phase plane inside and outside the limit cycle, respectively. The limit cycles are shown in Fig. 2 for a typical CME oscillator.

For relaying purposes, it is necessary to choose the sets $\{A_k\}$ to be indicative of

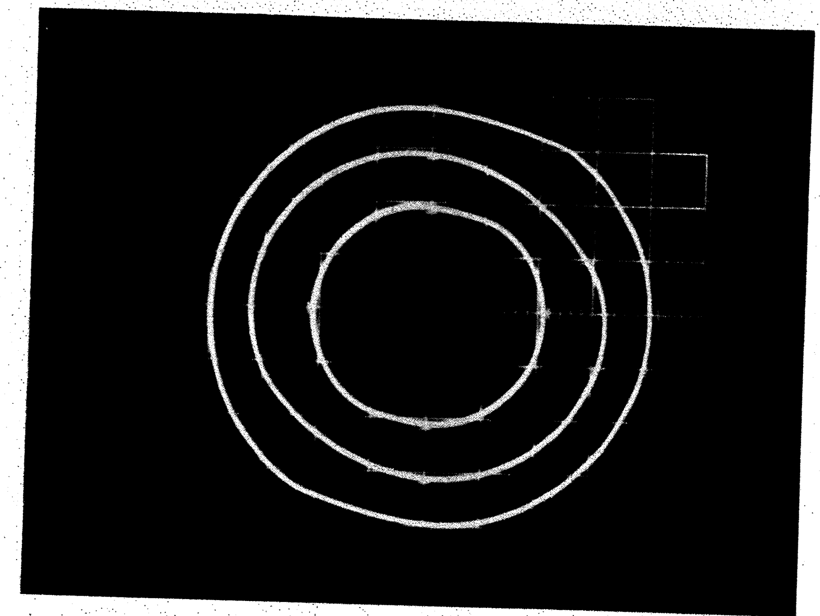

Figure 2. Multiple exposure photograph showing three stable limit cycles of CME, for $N = 5$, $E_1^2 = 0{\cdot}75$, $E_2^2 = 1{\cdot}85$, $E_3^2 = 2{\cdot}25$, $E_4^2 = 2{\cdot}65$, $E_5^2 = 3{\cdot}72$ V. The oscilloscope scale is 0·5 V/division.

particular events reflected by the actuation signal. To demonstrate a concrete practical situation, N is chosen equal to 5 and therefore there are three stable cycles for $k = 1, 3, 5$. Choosing A_1 to be the acceptable set requires that the set A_1 contains all initial conditions, X_0, such that sampled $f(t)$ and $f'(t)$ reflect conditions belonging to an acceptable set. A set is called acceptable if it is the set of initial conditions that result when the actuation signal is produced by events within the desirable realm of operating conditions. The sets A_3 and A_5 can be thought of as corresponding to 'moderate' and 'severe' hazardous situations (overloads or unacceptably fast changes). The relaying engineer must decide the location of these sets in the phase plane.

The remaining blocks of the MSD are conventional circuits. To have time delay before response, depending on the location of the initial conditions in the phase plane, the amplitude detector and delay block are introduced. The delay block provides a delay function, $\tau(E_k - E_1)$ $(k = 1, 3, 5)$, where $\tau(E_5 - E_1) < \tau(E_3 - E_1) < \tau(0) = \infty$. The amplitude detector, which produces $(E_k - E_1)$, is biased to ensure a negligible output when the initial conditions are in the acceptable set, and hence no response is appropriate.

4. Conclusions

The proposed device functions desirably when it is necessary to respond in different discrete ways to continuous actuation events. Further, responses to actuation events are based on both the actuation signal and its derivative, which in turn gives more effective control.

Reference

Saet, Y. S., and Viviani, G. L., 1984, Multi-stable periodical devices with variations on the theme of Van der Pol. *J. Franklin Inst.*, **318**, No. 6, 373–382.